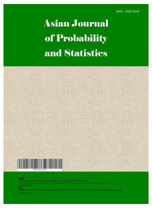

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Second Order Slope Rotatable Designs under Tri-diagonal Correlation Structure of Errors Using a Pair of Incomplete Block Designs

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Authors' contributions

This work was carried out in collaboration between both authors. Author BS designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author BRV managed the analyses of the study and the literature searches. Both authors read and approved the final manuscript.

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Abstract

Box and Hunter [1] introduced the concept of rotatability for response surface designs. The concept of slope-rotatability was introduced by Hader and Park [2] as an analogous to rotatability property, which is an important design criterion for response surface design. Slope-rotatable design is that of which the variance of partial derivative is a function of distance from the design (d). Recently, a few measures of slope-rotatability for a given response surface design was introduced. In this paper, a new method of slope rotatability for second order response surface designs under tri-diagonal correlation structure of errors using a pair of symmetrical unequal block arrangements with two unequal block sizes is studied. Further, a study on the dependence of variance function of the second order response surface at different design points for different values of tri-diagonal correlation coefficient ρ which lies between -0.9 to 0.9 and the distance from centre (d) is suggested.

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Keywords: Response surface designs; slope rotatability; tri-diagonal correlation errors; symmetrical unequal block arrangements with two unequal block sizes.

1 Introduction

Rotatable designs were introduced by Box and Hunter [1] for the exploration of response surface designs. Das and Narasimham [3] developed rotatable designs using balanced incomplete block designs (BIBD). Narasimham et al. [4] constructed SORD through a pair of incomplete block designs. Robust second order rotatable designs (RSORD) were introduced and studied by Das [5,6,7]. Rajyalakshmi and Victorbabu [8] studied rotatability for second order response surface designs under tri-diagonal correlation structure of errors using incomplete block designs (IBD), Victorbabu and Chiranjeevi [9] examined measure of degree of rotatability for second order response surface designs using symmetrical unequal block arrangements (SUBA) with two unequal block sizes .

Slope rotatable central composite designs introduced by Hader and Park [2]. Victorbabu and Narasimham [10,11] developed second order slope rotatable designs (SOSRD) utilizing BIBD and a pair of IBD respectively. Victorbabu and Narayanarao [12] constructed SOSRD utilizing a pair of symmetrical unequal block arrangements (SUBA) with two unequal block sizes. Victorbabu [13] suggested a review on SOSRD. Das [14] inroduced slope rotatability for second order response surface designs with correlated errors. Rajyalashmi [15] studied second order rotatable and slope rotatable designs under different correlated error structures, Rajyalakshmi and Victorbabu [16] constructed slope rotatability for second order response surface designs for under intra-class correlation structure of errors using central composite designs (CCD), Rajyalakshmi and Victorbabu [17,18,19,20] studied slope rotatability for second order response surface designs under tri-diagonal correlation structure of errors using central composite designs, pairwise balanced designs, SUBA with two unequal block sizes and BIBD respectively. Sulochana and Victorbabu [21] studied slope rotatability for second order response surface designs under tri-diagonal correlation structure of errors using a pair of BIBD.

In this paper, following the works of Victorbabu and Narasimham [10,11], Victorbabu and Narayanarao [12], Das [14], Rajyalakshmi [15], Rajyalakshmi and Victorbabu [8,16,17,18,19,20] here a study of slope rotatability for second order response surface designs under tri-diagonal correlation structure of errors using a pair of SUBA with two unequal block sizes is studied. Further we study the variance function of the estimated slopes for different values of tri-diagonal correlation coefficient ρ which lies between -0.9 to 0.9 and the distance from centre (d) is suggested.

2 Conditions for Second Order Slope Rotatable Designs under Tri-Diagonal Correlation Structure of Errors (cf. Das [14], Rajyalakshmi [15], Rajyalakshmi and Victorbabu [8,16,17,18,19,20]

A second order response surface design D = $((x_{iu}))$ for fitting,

$$Y_u(x)=\beta_0+\sum_{i=1}^{v}\beta_i X_{iu}+\sum_{i=1}^{v}\beta_{ii}X_{iu}^2+\sum_{i=1}^{v}\sum_{i<j}^{v}\beta_{ij}X_{iu}X_{ju}+e_u \tag{2.1}$$

where X_{iu} denotes the level of the i^{th} factor (i=1,2,…,v) in the u^{th} run (u=1,2,…,2n) of the experiment, e_u's are correlated random errors, is said to be slope rotatability for second order response surface designs under tri-diagonal correlation structure of errors, if the variance of the estimate of first order partial derivative of $Y_u(X_{1u},X_{2u},X_{3u},...,X_{vu})$ with respect to each independent variable (X_i) is only a function of the distance $\left(d^2=\sum_{i=1}^{v} X_i^2\right)$ of the point $\left(X_{1u}, X_{2u}, X_{3u}, \ldots, X_{vu}\right)$ from the origin (centre of the design). i.e, $V\left(\frac{\partial \hat{Y}_u}{\partial x_i}\right)=h\left(d^2\right)$. Such a spherical variance function $h\left(d^2\right)$ for estimation of slopes in the second order response surface is achieved if the design points satisfy the following conditions (cf. Das [5,7,14], (Rajyalakshmi [15]).

Following Box and Hunter [1], Hader and Park [2], Victorbabu and Narasimham [11], Das [14], Rajyalakshmi [15], Rajyalakshmi and Victorbabu [8,16,17,18,19,20] the general conditions for second order slope for rotatability under tri-diagonal correlation structure of errors can be obtained as follows. To simplify the fit of the second order response surface from design points D utilizing the method of least squares, the following simple symmetry conditions on D to facilitate easy solutions of the normal equations were imposed.

$$\sum_{u=1}^{2n} X_{iu}=0, \sum_{u=1}^{2n} X_{iu} X_{ju}=0, \sum_{u=1}^{2n} X_{iu} X_{ju}^2=0, \sum_{u=1}^{2n} X_{iu} X_{ju} X_{ku}=0, \sum_{u=1}^{2n} X_{iu}^3=0,$$

$$\sum_{u=1}^{2n} X_{iu} X_{ju}^3=0, \sum_{u=1}^{2n} X_{iu} X_{ju} X_{ku}^2=0, \sum_{u=1}^{2n} X_{iu} X_{ju} X_{ku} X_{lu}=0, \text{ for } i \neq j \neq k \neq l; \tag{2.2}$$

$$\sum_{u=1}^{2n} X_{iu}^2=\text{constant}=2n\gamma_2, \text{ for all } i, \tag{2.3}$$

$$\sum_{u=1}^{2n} X_{iu}^4=\text{constant}= c2n\gamma_4, \text{ for all } i, \tag{2.4}$$

$$\sum_{u=1}^{2n} X_{iu}^2 X_{ju}^2=\text{constant}=2n\gamma_4, \text{ for all values } i \neq j \tag{2.5}$$

From (2.4) and (2.5), we have,

$$\sum_{u=1}^{2n} X_{iu}^4= c \sum_{u=1}^{2n} X_{iu}^2 X_{ju}^2 \tag{2.6}$$

where c, γ_2 and γ_4 are constants. The summation is over the designs points, and ρ be the correlation coefficient.

The variances and covariances of the estimated parameters under the tri-diagonal correlation structure of errors are as given below.

$$V\left(\hat{\beta}_0\right)=\frac{\gamma_4 (c+v-1)(1+\rho)\sigma^2}{2n\Delta} \tag{2.7}$$

$$V\left(\hat{\beta}_i\right)=\frac{\sigma^2(1-\rho^2)}{2n\gamma_2} \tag{2.8}$$

$$V\left(\hat{\beta}_{ij}\right)=\frac{\sigma^2(1-\rho^2)}{2n\gamma_4} \tag{2.9}$$

$$V\left(\hat{\beta}_{ii}\right)=\frac{\sigma^2(1-\rho^2)[\gamma_4\left(c+v-2\right)-(v-1)\gamma_2^2(1-\rho)]}{(c-1)(2n)\gamma_4\Delta} \tag{2.10}$$

$$Cov\left(\hat{\beta}_0,\hat{\beta}_{ii}\right)=\frac{-\sigma^2\gamma_2^2(1-\rho^2)}{2n\Delta} \tag{2.11}$$

$$Cov\left(\hat{\beta}_{ii},\hat{\beta}_{ij}\right)=\frac{\sigma^2(1-\rho^2)[\gamma_2^2\left(1-\rho\right)-\gamma_4]}{(c-1)(2n)\gamma_4\Delta} \tag{2.12}$$

Where $\Delta=\left[\gamma_4\left(c+v-1\right)-v\gamma_2^2\left(1-\rho\right)\right]$ and the other covariances are zero.

An inspection of the variance of $\hat{\beta}_0$ shows that a necessary condition for the existence of a non-singular slope rotatability for second order response surface designs under tri-diagonal correlation structure is

$$\left[\gamma_4\left(c+v-1\right)-v\gamma_2^2\left(1-\rho\right)\right]>0 \tag{2.13}$$

From (2.13), we have,

$$\frac{\gamma_4}{\gamma_2^2}>\frac{v(1-\rho)}{c+v-1}\text{ (non-singularity condition)} \tag{2.14}$$

The condition (2.14) exists then only the design exists.

For the second order model,

$$\frac{\partial\hat{Y}}{\partial X_i}=\hat{\beta}_i+2\hat{\beta}_{ii}X_i+\sum_{i=1,j\neq i}^{v}\hat{\beta}_{ij}X_j,$$

$$V\left(\frac{\partial\hat{Y}}{\partial X_i}\right)=V\left(\hat{\beta}_i\right)+4x_i^2V\left(\hat{\beta}_{ii}\right)+\sum_{i=1,j\neq i}^{v}X_j^2V\left(\hat{\beta}_{ij}\right) \tag{2.15}$$

The condition for right hand side of equation (2.15) to be a function of the distance $(d^2=\sum_{i=1}^{v}X_i^2)$ alone (for second order slope rotatability) is,

$$V\left(\hat{\beta}_{ii}\right)=\frac{1}{4}V\left(\hat{\beta}_{ij}\right) \tag{2.16}$$

On simplification of (2.16) using (2.9) and (2.10) leads to

$$\gamma_4\left[v(5\text{-}c)\text{-}(c\text{-}3)^2\right]+\gamma_2^2\left[v(c\text{-}5)+4\right]\left(1-\rho\right)=0 \tag{2.17}$$

For $\rho=0$, equation (2.17) reduces to

$$\gamma_4\left[v(5\text{-}c)\text{-}(c\text{-}3)^2\right]+\gamma_2^2\left[v(c\text{-}5)+4\right]=0 \tag{2.18}$$

Equation (2.18) is similar to slope rotatability condition for second order response surface designs suggested by Victorbabu and Narasimham [10].

Therefore, equations (2.2) to (2.12), (2.14) to (2.17) give a set of conditions for slope rotatability for second order response surface designs under tri-diagonal correlation structure of errors for any general second order response surface design. Further,

$$V\left(\frac{\partial\hat{Y}}{\partial X_i}\right)=\frac{1\text{-}\rho^2}{N}\left(\frac{1}{\gamma_2}+\frac{d^2}{\gamma_4}\right)\sigma^2 \tag{2.19}$$

3 Slope Rotatability for Second Order Response Surface Designs under Tri-diagonal Correlation Structure of Errors Using a Pair of SUBA with Two Unequal Block Sizes

Following the methods of constructions of Das [5,7,14], Victorbabu and Narasimham [10,11], Rajyalakshmi [15], Rajyalakshmi and Victorbabu [8,16,17,18,19,20], here a study on slope rotatability for second order response surface designs under tri-diagonal correlation structure of errors using a pair of SUBA with two unequal block sizes is studied. Let ρ be the correlation errors of any two observations.

Define $D_1=\left(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1\right)$, $k_1=\sup(k_{11},k_{12})$, $b_{11}+b_{12}=b_1$ with $r_1 \lesseqgtr c\lambda_1$ and $D_2=\left(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2\right)$, $k_2=\sup(k_{21},k_{22})$, $b_{21}+b_{22}=b_2$ with $r_2 \gtreqless c\lambda_2$ be two SUBA with two unequal block sizes respectively. Let $2^{t(k_1)}$ and $2^{t(k_2)}$ denotes resolution-V fractional factorial design replicate of 2^{k_1} and 2^{k_2} in ± 1 levels (cf. Narasimham et al. [4]). The design points achieved from the transpose of incidence matrix of the design D_1 by multiplication are denoted by $\left[1-\left(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1\right)\right]2^{t(k_1)}$. Let $\left[1-\left(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1\right)\right]2^{t(k_1)}$ are the $b_1 2^{t(k_1)}$ design points achieved from D_1 by multiplication [22]). $\left[a-\left(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2\right)\right]2^{t(k_2)}$ are the $b_2 2^{t(k_2)}$ design points achieved from D_2 by multiplication. Consider n_o central points (0, 0,..., 0) in the design. Taking the method of construction of slope rotatability for second order response surface designs using a pair of SUBA with two unequal block sizes (cf. Victorbabu and Narayanarao [12]) having n $\left(n=b_1 2^{t(k_1)}+b_2 2^{t(k_2)}\right)$ non-central design points. The n non-central design points $\left(n=b_1 2^{t(k_1)}+b_2 2^{t(k_2)}\right)$

are extended to 2n (=N) design points by adding n $(n_0=n)$ central points (0, 0,…, 0) just below or above the non- central design points. Here, the total number of design points of the slope rotatability for second order response surface designs under tri-diagonal correlation structure of errors using a pair of SUBA with two unequal block sizes are 2n. The method of construction of slope rotatability for second order response surface designs under tri-diagonal correlation structure of errors is given below.

Theorem 3.1

Consider the design points,

$\left[1\text{-}\left(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1\right)\right]2^{t(k_1)}\cup\left[a\text{-}\left(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2\right)\right]2^{t(k_2)}\cup n_0$ give a v-dimensional slope rotatability for second order response surface design under tri-diagonal correlation structure of errors using a pair of SUBA with two unequal block sizes in $N=2n=n+n_0$ design points, where a^2 is a positive real root of the biquadratic equation,

$$\begin{aligned}
&\left[2^{2t(k_2)}N\left\{r_2\lambda_2\left(6\text{-}v\right)\text{-}r_2^2+\lambda_2^2\left(5v\text{-}9\right)\right\}+\left(2^{3t(k_2)}r_2^2\left(vr_2\text{-}5v\lambda_2+4\lambda_2\right)\left(1\text{-}\rho\right)\right)\right]a^8+\\
&\left[2^{t(k_1)+2t(k_2)+1}r_1r_2\left(vr_2\text{-}5v\lambda_2+4\lambda_2\right)\left(1\text{-}\rho\right)\right]a^6+\\
&\left[\begin{array}{l}2^{t(k_1)+t(k_2)}N\left\{\lambda_1\lambda_2\left(10v\text{-}18\right)+\left(6\text{-}v\right)\left(r_1\lambda_2+r_2\lambda_1\right)\text{-}2r_1r_2\right\}+\\ \left(2^{t(k_1)+2t(k_2)}r_2^2\left(vr_1\text{-}5v\lambda_1+4\lambda_1\right)+2^{2t(k_1)+t(k_2)}r_1^2\left(vr_2\text{-}5v\lambda_2+4\lambda_2\right)\right)\left(1\text{-}\rho\right)\end{array}\right]a^4+\\
&\left[2^{2t(k_1)+t(k_2)+1}r_1r_2\left(vr_1\text{-}5v\lambda_1+4\lambda_1\right)\left(1\text{-}\rho\right)\right]a^2+\\
&\left[2^{2t(k_1)}N\left\{\lambda_1^2\left(5v\text{-}9\right)+(6\text{-}v)r_1\lambda_1\text{-}r_1^2\right\}+2^{3t(k_1)}r_1^2\left(vr_1\text{-}5v\lambda_1+4\lambda_1\right)\left(1\text{-}\rho\right)\right]=0
\end{aligned} \tag{3.1}$$

If at least one positive real root a^2 exists in (3.1), then only the design exists.

Proof: For the design points generated from a pair of SUBA with two unequal block sizes, simply symmetry conditions (2.2) are true. Further, from (2.3) to (2.5) we have,

$$\sum_{u=1}^{2n}X_{iu}^2=r_1 2^{t(k_1)}+r_2 2^{t(k_2)}a^2=2n\gamma_2,\text{ for all i,} \tag{3.2}$$

$$\sum_{u=1}^{2n}X_{iu}^4=r_1 2^{t(k_1)}+r_2 2^{t(k_2)}a^4=c2n\gamma_4,\text{ for all i,} \tag{3.3}$$

$$\sum_{u=1}^{2n}X_{iu}^2X_{ju}^2=\lambda_1 2^{t(k_1)}+\lambda_2 2^{t(k_2)}a^4=2n\gamma_4,\text{ for all } i\neq j \tag{3.4}$$

From (3.2), (3.3) and (3.4) we get $\gamma_2=\dfrac{r_1 2^{t(k_1)}+r_2 2^{t(k_2)}a^2}{2n}$, $\gamma_4=\dfrac{\lambda_1 2^{t(k_1)}+\lambda_2 2^{t(k_2)}a^4}{2n}$ and $c=\dfrac{r_1 2^{t(k_1)}+r_2 2^{t(k_2)}a^4}{\lambda_1 2^{t(k_1)}+\lambda_2 2^{t(k_2)}a^4}$. Satisfying the equation (2.17) by simplification γ_2, γ_4 and c, we get (3.1).

Solving equation (3.1) we get the slope rotatability values (a) for second order response surface designs under tri- diagonal correlation structure of errors using a pair of SUBA with two unequal block sizes for different factors.

Example: We illustrate the above method with the construction of slope rotatability for second order response surface designs under tri- diagonal correlation structure of errors for v =12 factors with the help of a pair of SUBA with two unequal block sizes with parameters

$$D_1=\left(v=12,\ b_1=13,\ r_1=4,\ k_{11}=3,\ k_{12}=4,\ b_{11}=4,\ b_{12}=9,\ \lambda_1=1\right),$$

$$D_2=\left(v=12,\ b_2=26,\ r_2=6,\ k_{21}=2,\ k_{22}=3,\ b_{21}=6,\ b_{22}=20,\ \lambda_2=1\right) \text{ is given below.}$$

The design points,

$$\left[1\text{-}\left(v=12,\ b_1=13,\ r_1=4,\ k_{11}=3,\ k_{12}=4,\ b_{11}=4,\ b_{12}=9,\ \lambda_1=1\right)\right]2^4U$$
$$\left[a\text{-}\left(v=12,\ b_2=26,\ r_2=6,\ k_{21}=2,\ k_{22}=3,\ b_{21}=6,\ b_{22}=20,\ \lambda_2=1\right)\right]2^3U(n_0=416)$$

will give a slope-rotatability for second order response surface design under tri-diagonal correlation structure of errors in N = 832 design points. From (3.2) to (3.4) we have,

$$\sum_{u=1}^{2n} X_{iu}^2=64+48a^2=2n\gamma_2 \tag{3.5}$$

$$\sum_{u=1}^{2n} X_{iu}^4=64+48a^4=c2n\gamma_4 \tag{3.6}$$

$$\sum_{u=1}^{2n} X_{iu}^2X_{ju}^2=16+8a^4=2n\gamma_4 \tag{3.7}$$

From (3.5), (3.6) and (3.7), we get $\gamma_2=\dfrac{4+3a^2}{52}$, $\gamma_4=\dfrac{2+a^4}{104}$ and $c=\dfrac{8+6a^4}{2+a^4}$. Substituting γ_2, γ_4 and c in (2.17) and on simplification, we get the following biquadratic equation in a^2.

$$\left[72\rho+201\right]a^8-\left[192(1-\rho)\right]a^6+\left[56\rho+100\right]a^4+\left[192(1-\rho)\right]a^2-\left[128\rho+444\right]=0 \tag{3.8}$$

Alternatively from equation (3.1) we obtain equation (3.8) directly. Solving (3.8), we get a=1.080191 (by taking $\rho=0.1$). Substituting this value of 'a' in (3.5), (3.6) and (3.7) and on simplification we obtain $\gamma_2=0.1442, \gamma_4=0.0323$ and c=4.81. Non-singularity condition (2.14) is also satisfied as 1.5534 > 0.6831. From (2.7) to (2.12), we can obtain the variances and covariances. Further, from (2.19), we get

$$V\left(\frac{\partial \hat{Y}}{\partial X_i}\right)=\left(0.0083+0.0372d^2\right)\left(1\text{-}\rho^2\right)\sigma^2 \tag{3.9}$$

The variance of estimated slopes of these slope rotatability for second order response surface designs under tri- diagonal correlation structure of errors for tri-diagonal correlation coefficient ρ lies between -0.9 to 0.9 for v = 12 factors are given in Table 1.

Table 1. The variances of estimated derivatives (slopes) for v = 12 factors

$D_1 = (v{=}12,\ b_1{=}13,\ r_1{=}4,\ k_{11}{=}3,\ k_{12}{=}4,\ b_{11}{=}4,\ b_{12}{=}9,\ \lambda_1{=}1)$,

$D_2 = (v{=}12,\ b_2{=}26,\ r_2{=}6,\ k_{21}{=}2,\ k_{22}{=}3,\ b_{21}{=}6,\ b_{22}{=}20,\ \lambda_2{=}1)$

n = 416, N = 832

ρ	**a**	$V\left(\partial \hat{Y}_u / \partial x_i\right)$
-0.9	1.476849	$0.001126\ \sigma^2 + 0.003515\ d^2\ \sigma^2$
-0.8	1.372078	$0.002332\ \sigma^2 + 0.008116\ d^2\ \sigma^2$
-0.7	1.284725	$0.003561\ \sigma^2 + 0.013494\ d^2\ \sigma^2$
-0.6	1.221019	$0.004721\ \sigma^2 + 0.01894\ d^2\ \sigma^2$
-0.5	1.1776	$0.005744\ \sigma^2 + 0.023897\ d^2\ \sigma^2$
-0.4	1.147717	$0.006602\ \sigma^2 + 0.028111\ d^2\ \sigma^2$
-0.3	1.126323	$0.007286\ \sigma^2 + 0.031515\ d^2\ \sigma^2$
-0.2	1.110359	$0.007794\ \sigma^2 + 0.034091\ d^2\ \sigma^2$
-0.1	1.098018	$0.008123\ \sigma^2 + 0.035832\ d^2\ \sigma^2$
0	1.088195	$0.008275\ \sigma^2 + 0.03674\ d^2\ \sigma^2$
0.1	1.080191	$0.008249\ \sigma^2 + 0.036814\ d^2\ \sigma^2$
0.2	1.073539	$0.008046\ \sigma^2 + 0.036055\ d^2\ \sigma^2$
0.3	1.067922	$0.007663\ \sigma^2 + 0.034463\ d^2\ \sigma^2$
0.4	1.063114	$0.007104\ \sigma^2 + 0.032038\ d^2\ \sigma^2$
0.5	1.058951	$0.006365\ \sigma^2 + 0.028779\ d^2\ \sigma^2$
0.6	1.055309	$0.005449\ \sigma^2 + 0.024689\ d^2\ \sigma^2$
0.7	1.052097	$0.004354\ \sigma^2 + 0.019766\ d^2\ \sigma^2$
0.8	1.049241	$0.003081\ \sigma^2 + 0.014009\ d^2\ \sigma^2$
0.9	1.046685	$0.001629\ \sigma^2 + 0.007421\ d^2\ \sigma^2$

Table 2. A study of dependence of estimated slope second order response surface design under tri-diagonal correlation structure of errors using a pair of SUBA with two unequal block sizes at different design points for v =12 factors for different values of ρ, d and σ=1 is given below

	d=0.1	**d=0.2**	**d=0.3**	**d=0.4**	**d=0.5**	**d=0.6**	**d=0.7**	**d=0.8**	**d=0.9**	**d=1**
-0.9	0.001161	0.001266	0.001442	0.001688	0.002005	0.0023914	0.002848	0.003376	0.003974	0.004641
-0.8	0.002413	0.002657	0.003062	0.003631	0.004361	0.005254	0.006308	0.007526	0.008906	0.010448
-0.7	0.003696	0.004101	0.004775	0.00572	0.006935	0.008419	0.010173	0.012197	0.014491	0.017055
-0.6	0.00491	0.005479	0.006426	0.007752	0.009457	0.011541	0.014004	0.016845	0.020066	0.023666
-0.5	0.005983	0.006699	0.007895	0.009568	0.011718	0.014369	0.017453	0.021038	0.025101	0.029641
-0.4	0.006883	0.007726	0.009132	0.011099	0.013629	0.016722	0.020376	0.024593	0.029372	0.034713
-0.3	0.007601	0.008546	0.010122	0.01233	0.015165	0.018631	0.022728	0.027456	0.032813	0.038801
-0.2	0.008134	0.009158	0.010862	0.013249	0.016317	0.020066	0.024499	0.029612	0.035408	0.041885
-0.1	0.008481	0.009556	0.011348	0.013856	0.017081	0.021021	0.025679	0.031054	0.037145	0.043953
0	0.008642	0.009744	0.011582	0.014153	0.01746	0.021501	0.026278	0.031788	00.038034	0.045105
0.1	0.008617	0.009722	0.011562	0.014139	0.017453	0.021502	0.026287	0.031809	0.038068	0.045063
0.2	0.008406	0.009488	0.011291	0.013814	0.017059	0.021026	0.025713	0.031121	0.037251	0.044101
0.3	0.008007	0.009042	0.010764	0.013177	0.016279	0.020069	0.024549	0.029719	0.035578	0.042126
0.4	0.007424	0.008386	0.009874	0.012231	0.015114	0.018638	0.022803	0.027608	0.033055	0.039142
0.5	0.006653	0.007516	0.008955	0.010969	0.013559	0.016725	0.020467	0.024784	0.029676	0.035144
0.6	0.005696	0.006437	0.007679	0.009399	0.011621	0.014337	0.017547	0.021249	0.025447	0.030138
0.7	0.004552	0.005145	0.006133	0.007517	0.009296	0.011469	0.014039	0.017004	0.020364	0.02412
0.8	0.003221	0.003641	0.004342	0.005322	0.006583	0.008124	0.009945	0.012047	0.014428	0.01709
0.9	0.001703	0.001926	0.002297	0.002816	0.003484	0.004301	0.005265	0.006378	0.00764	0.00905

4 A Study of Dependence of Variance Function of the Second Order Response Surface at Different Design Points

Here, we study the dependence of variance function of response at given design points of slope rotatability for second order response surface designs under tri- diagonal correlation structure of errors using a pair of SUBA with two unequal block sizes. Given v factors different values of tri-diagonal correlation coefficient ρ and distance from centre d, the variances are tabulated between 0 and 1.

In equation (3.9), if we take, ρ =0.1, d=0.1 and σ = 1, we get,

$$V\left(\frac{\partial \hat{Y}}{\partial X_i}\right) = 0.0086.$$

The numerical calculations are appended in Table 2.

5 Conclusions

From Tables 1 and 2 we observe that

1. Suppose the values of ρ increases, slope rotatability value is decreases.
2. For $\rho=0$, the variance of estimated derivatives of slope rotatability for second order response surface designs under tri- diagonal correlation structure of errors is equal to the slope rotatability for second order response surface designs uncorrelated errors.
3. For given v and ρ, the variance of the estimated slope increases as d increases.
4. For given v and d, the variance of the estimated slope decreases as ρ increases.

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Competing Interests

Authors have declared that no competing interests exist.

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