



Exact Analytical Solution of Ivancevic Options Pricing Model (IOPM) or Schrödinger's Equation via ADM and SBA Methods

Kamate Adama ^a, Pare Daouda ^a, Mouyedo Loufouilou Justin ^b
and Youssouf Pare ^{a*}

^a*Département de Mathématiques, Université Joseph KI-ZERBO, Burkina Faso.*

^b*Département de Mathématiques, Université Marien M'Gouabi, Congo Brazzaville.*

Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/ARJOM/2022/v18i330366

Open Peer Review History:

This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: <https://www.sdiarticle5.com/review-history/84786>

Received: 20 January 2022

Accepted: 24 March 2022

Published: 08 April 2022

Review Article

Abstract

This paper is devoted to the study of the general equation of the Ivancevic option pricing model (IOPM) or Schrödinger's equation and to determine its analytical solution via the methods of numerical analysis ADM and SBA. The Ivancevic option pricing model is an adaptive wave model that is a nonlinear wave alternative to the standard Black-Scholes option pricing model, it is also a model that links quantum mechanics and financial mathematics.

Keywords: Ivancevic or Shrodinger model; Adomian Method (ADM); SBA method; Succesive approximations.

2010 Mathematics Subject Classification: 35A25; 9324; 97I50; 44Axx.

1 Introduction

The classical Black-Scholes model (BSM) is an important financial model for option pricing an valuation. In this paper, we are interested in the determination of the analytical solution of the

**Corresponding author: E-mail: pareyoussof@gmail.com;*

general equation of the Ivancevic [1] or Schrödinger [2] model in quantum mechanics. It is about the equations:

$$(E) : \begin{cases} i \frac{\partial w(t, x)}{\partial t} + \varepsilon \frac{\partial^2 w(t, x)}{\partial x^2} + q |w(t, x)|^{2p} w(t, x) = 0; p \in \mathbb{N}^* \\ w(0, x) = \beta e^{iax} \end{cases}$$

and

$$(F) : \begin{cases} i \frac{\partial w(t, x)}{\partial t} + \varepsilon \frac{\partial^2 w(t, x)}{\partial x^2} + \mu w(t, x) + q |w(t, x)|^{2p} w(t, x) = 0; p \in \mathbb{N}^* \\ w(0, x) = \beta e^{iax} \end{cases}$$

where $\varepsilon > 0, \mu > 0$ and $q > 0$.

2 Description of Numerical Method ADM and SBA

2.1 Numerical method ADM

Consider the functional equation below :

$$Fw = f \tag{2.1}$$

where F is an operator defined in the Hilbert space H in H , f is a given function in H and w is the unknown function. Let us decompose as follows

$$F = L - R - N \tag{2.2}$$

Where L is the linear part of inverse L^{-1} , R the linear remainder and N the nonlinear part, (2.1) becomes :

$$Lw - Rw - Nw = f \tag{2.3}$$

Applying L^{-1} to (2.3), we get the Adomian canonical form [3]:

$$w = \theta + L^{-1}f + L^{-1}Rw + L^{-1}Nw \tag{2.4}$$

where

$$L\theta = 0.$$

Let us determine the solution of (2.1) in the form of a convergent series [4]

$$w = \sum_{n=0}^{+\infty} w_n$$

and

$$Nw = \sum_{n=0}^{+\infty} A_n < +\infty$$

where the

$$A_n = A_n(w_0, w_1, \dots, w_n)$$

are Adomian polynomials [5]. We get the following Adomian algorithm [6]:

$$\begin{cases} w_0 = \theta + L^{-1}f \\ w_{n+1} = L^{-1}Rw_n + L^{-1}A_n; n \geq 0. \end{cases}$$

2.2 The Adomian polynomials

Definition 2.1. The Adomian polynomials are defined by:

$$\begin{cases} A_0 = N(w_0) \\ A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} [N(\sum_{k=0}^{+\infty} \lambda^k w_k)]_{\lambda=0} : n \geq 1 \end{cases}$$

Theorem 2.1. The Adomian polynomials are calculated using the formula:

$$\left[\frac{d^n}{d\lambda^n} \sum_{k=0}^n \lambda^k A_k \right]_{\lambda=0} = \left[\frac{d^n}{d\lambda^n} N \left(\sum_{k=0}^n \lambda^k w_k \right) \right]_{\lambda=0}$$

2.3 Numerical method SBA

Consider the functional equation below:

$$Fw = f \tag{2.5}$$

where F is an operator defined in the Hilbert space H in H , f is a given function in H and w is the unknown function. Let us decompose as follows

$$F = L - R - N \tag{2.6}$$

Where L is the linear part of inverse L^{-1} , R the linear remainder and N the nonlinear part, (2.1) becomes:

$$Lw - Rw - Nw = f \tag{2.7}$$

Applying L^{-1} to (2.3), we get the Adomian canonical form:

$$w = \theta + L^{-1}f + L^{-1}Rw + L^{-1}Nw \tag{2.8}$$

where

$$L\theta = 0.$$

Equation (2.5) is the Adomian canonical form [7]. Using the successive approximations [1], we get:

$$w^k = \theta + L^{-1}(f) + L^{-1}(R(w^k)) + L^{-1}(N(w^{k-1})); k \geq 1 \tag{2.9}$$

This let's to the following Adomian algorithm:

$$\begin{cases} w_0^k = \theta + L^{-1}(f) + L^{-1}(N(w^{k-1})); k \geq 1 \\ w_n^k = L^{-1}(R(w_{n-1}^k)), n \geq 1 \end{cases} \tag{2.10}$$

The Picard principle is then applied to equation (2.7): let w^0 be such that $N(w^0) = 0$, for $k = 1$, we get:

$$\begin{cases} w_0^1 = \theta + L^{-1}(f) + L^{-1}(N(w^0)) \\ w_n^1 = L^{-1}(R(w_{n-1}^1)), n \geq 1 \end{cases} \tag{2.11}$$

If the series $\left(\sum_{n=0}^{\infty} w_n^1\right)$ converges, then $w^1 = \left(\sum_{n \geq 1} w_n^1\right)$

For $k = 2$, we get:

$$\begin{cases} w_0^2 = \theta + L^{-1}(f) + L^{-1}(N(w^1)) \\ w_n^2 = L^{-1}(R(w_{n-1}^2)), n \geq 1 \end{cases} \tag{2.12}$$

If the series $\left(\sum_{n=0}^{\infty} w_n^2\right)$ converges, then $w^2 = \left(\sum_{n \geq 0} w_n^2\right)$.

This process is repeated to k .

If the series $\left(\sum_{n=0}^{\infty} w_n^k\right)$ converges, then $w^k = \left(\sum_{n \geq 0} w_n^k\right)$.

Therefore $w = \lim_{k \rightarrow +\infty} w^k$ is the solution of the problem, with the following hypothese at the step k : $N(w^k) = 0, \forall k \geq 0$.

Theorem 2.2. Consider the following Cauchy problem:

$$(p) : \begin{cases} L_t w(t, x) = \varepsilon \Delta w(t, x) + \mu w(t, x) + Nw(t, x), (t, x) \in \Omega \\ w(0, x) = h(x) \end{cases}$$

Associated to the problem (p), the SBA allorithm is given as:

$$(p_{SBA}) : \begin{cases} w_0^k(t, x) = h(x) + L_t^{-1} [N(w^{k-1}(t, x))] ; k \geq 1 \\ w_{n+1}^k(t, x) = L_t^{-1} [\varepsilon \Delta w_n^k(t, x) + \mu w_n^k(t, x)] ; n \geq 0 \end{cases}$$

(H₁) : There is $w^0(t, x)$ at the step $k = 1$, such as $Nw^0(t, x) = 0$.

(H₂) : At the step $k = 1$, $w^1(t, x)$ is the solution of :

$$\begin{cases} w_0^1(t, x) = h(x) \\ w_{n+1}^1(t, x) = L_t^{-1} [\varepsilon \Delta w_n^1(t, x) + \mu w_n^1(t, x)] ; n \geq 0. \end{cases}$$

(H₃) : At the step $k = 2$, $Nw^1(t, x) = 0$. So the algorithm :

$$(p_{SBA}) : \begin{cases} w_0^k(t, x) = h(x) + L_t^{-1} [N(w^{k-1}(t, x))] ; k \geq 1 \\ w_{n+1}^k(t, x) = L_t^{-1} [\varepsilon \Delta w_n^k(t, x) + \mu w_n^k(t, x)] ; n \geq 0 \end{cases}$$

is convergent for $k \geq 2$ and we obtain : $w^1(t, x) = w^2(t, x) = \dots = w^k(t, x)$.

From which the unique solution of the problem $w(t, x) = \lim_{k \rightarrow +\infty} w^k(t, x)$.

Proof. At step $k = 1$, we have the following algorithm:

$$(p_1) \begin{cases} w_0^1(t, x) = h(x) \\ w_{n+1}^1(t, x) = L_t^{-1} [\varepsilon \Delta w_n^1(t, x) + \mu w_n^1(t, x)] ; n \geq 0 \end{cases}$$

according to hypothesis (H₁) and (H₂), the solution of (p₁) is $w^1(t, x) = \sum_{n=0}^{+\infty} w_n^1(t, x)$. According to the hypothesis (H₃), at step $k = 2$, $Nw^1(t, x) = 0$

we get the following alorthm:

$$(p_2) \begin{cases} w_0^2(t, x) = h(x) \\ w_{n+1}^2(t, x) = L_t^{-1} [\varepsilon \Delta w_n^2(t, x) + \mu w_n^1(t, x)] ; n \geq 0 \end{cases}$$

Thus, we obtain the same algorithm as in step $k = 1$, then $w^2(t, x) = w^1(t, x)$. Thus, in a recursive way it will be for each step $k \geq 2$, $w^1(t, x) = w^2(t, x) = w^3(t, x) = \dots$

Then the solution of the problem (p) is $w(t, x) = \lim_{k \rightarrow +\infty} w^k(t, x)$.

Suppose that the problem (p) has two distinct solutions $w(t, x) \neq v(t, x)$, and consider their difference $\varphi(t, x) = w(t, x) - v(t, x)$.

For each solution, we have:

$$\begin{cases} w_0^k(t, x) = h(x) + L_t^{-1} [N(w^{k-1}(t, x))] ; k \geq 1 \\ w_{n+1}^k(t, x) = L_t^{-1} [\varepsilon \Delta w_n^k(t, x) + \mu w_n^k(t, x)] ; n \geq 0 \end{cases}$$

and

$$\begin{cases} v_0^k(t, x) = h(x) + L_t^{-1} [N(v^{k-1}(t, x))] ; k \geq 1 \\ v_{n+1}^k(t, x) = L_t^{-1} [\varepsilon \Delta v_n^k(t, x) + \mu v_n^k(t, x)] ; n \geq 0 \end{cases}$$

$\forall k \geq 1, N(w^{k-1}(t, x)) = 0$ and $N(v^{k-1}(t, x)) = 0$, so we obtain:

$$\begin{cases} w_0^k(t, x) = h(x) ; k \geq 1 \\ w_{n+1}^k(t, x) = L_t^{-1} [\varepsilon \Delta w_n^k(t, x) + \mu w_n^k(t, x)] ; n \geq 0 \end{cases}$$

and

$$\begin{cases} v_0^k(t, x) = h(x) ; k \geq 1 \\ v_{n+1}^k(t, x) = L_t^{-1} [\varepsilon \Delta v_n^k(t, x) + \mu v_n^k(t, x)] ; n \geq 0 \end{cases}$$

For the difference we get:

$$\begin{cases} \varphi_0^k(t, x) = 0 ; k \geq 1 \\ \varphi_{n+1}^k(t, x) = L_t^{-1} [\varepsilon \Delta \varphi_n^k(t, x) + \mu \varphi_n^k(t, x)] ; n \geq 0 \end{cases}$$

from which

$$\begin{cases} \varphi_0^k(t, x) = 0 \\ \varphi_1^k(t, x) = 0 \\ \dots \\ \varphi_n^k(t, x) = 0, \forall n \geq 0 \end{cases}$$

Thus $\varphi_n^k(t, x) = \sum_{n=0}^{+\infty} \varphi_n^k(t, x) = 0$ and $w(t, x) = v(t, x)$ which contradicts our hypothesis. Therefore the problem (p) has a unique solution $w(t, x)$. □

3 Resolution via Numerical Method ADM

Consider the following equation

$$(E) : \begin{cases} i \frac{\partial w(t, x)}{\partial t} + \varepsilon \frac{\partial^2 w(t, x)}{\partial x^2} + q |w(t, x)|^{2p} w(t, x) = 0 \\ w(0, x) = \beta e^{iax} \end{cases}$$

Let us determine the canonical form of Adomian, the equation

$$i \frac{\partial w(t, x)}{\partial t} + \varepsilon \frac{\partial^2 w(t, x)}{\partial x^2} + q |w(t, x)|^{2p} w(t, x) = 0; p \in \mathbb{N}^*$$

is equivalent to

$$\frac{\partial w(t, x)}{\partial t} = i\varepsilon \frac{\partial^2 w(t, x)}{\partial x^2} + iq |w(t, x)|^{2p} w(t, x)$$

from which we obtain the canonical form:

$$w(t, x) = w(0, x) + i\varepsilon \int_0^t \frac{\partial^2 w(z, x)}{\partial x^2} dz + iq \int_0^t |w(z, x)|^{2p} w(z, x) dz.$$

Thus, we obtain the Adomian algorithm:

$$\begin{cases} w_0(t, x) = w(0, x) \\ w_{n+1}(t, x) = i\varepsilon \int_0^t \frac{\partial^2 w_n(z, x)}{\partial x^2} dz + iq \int_0^t A_n(z, x) dz; n \geq 0 \end{cases}$$

Let's calculate the polynomials: A_0, A_1, A_2, \dots

$$\begin{cases} A_0 = |\beta|^{2p} w_0 \\ A_1 = w_1 (w_0 \bar{w}_0)^p + pw_0 (w_1 \bar{w}_0 + \bar{w}_1 w_0)^{p-1} \\ A_2 = 2(aw_0)^p w_2 + 2p(aw_1 + bw_0)^{p-1} (w_1) + p(p-1) (2(aw_2 + bw_1 + cw_0))^{p-2} (w_0) \\ \dots \end{cases}$$

Let's calculate the terms: $w_0(t, x), w_1(t, x), \dots$

we obtain thus:

$$\begin{cases} w_0(t, x) = \beta e^{iax} \\ w_1(t, x) = \beta it (-\varepsilon a^2 + q |\beta|^{2p}) e^{iax} \\ w_2(t, x) = \beta \frac{[it (-\varepsilon a^2 + q |\beta|^{2p})]^2}{2!} e^{iax} \\ w_3(t, x) = \beta \frac{[it (-\varepsilon a^2 + q |\beta|^{2p})]^3}{3!} e^{iax} \\ \dots \\ w_n(t, x) = \beta \frac{[it (-\varepsilon a^2 + q |\beta|^{2p})]^n}{n!} e^{iax} \end{cases}$$

Therefore, the solution of problem (E) obtained by the ADM method is :

$$w(t, x) = \sum_{n=0}^{+\infty} w_n(t, x) = \beta \exp [i ((-\varepsilon a^2 + q |\beta|^{2p}) t + ax)].$$

Consider the following equation

$$(F) : \begin{cases} i \frac{\partial w(t, x)}{\partial t} + \varepsilon \frac{\partial^2 w(t, x)}{\partial x^2} + \mu w(t, x) + q |w(t, x)|^{2p} w(t, x) = 0 \\ w(0, x) = \beta e^{iax} \end{cases}$$

Let us determine the canonical form of Adomian, the equation

$$i \frac{\partial w(t, x)}{\partial t} + \varepsilon \frac{\partial^2 w(t, x)}{\partial x^2} + \mu w(t, x) + q |w(t, x)|^{2p} w(t, x) = 0; p \in \mathbb{N}^*$$

is equivalent to

$$\frac{\partial w(t, x)}{\partial t} = i\varepsilon \frac{\partial^2 w(t, x)}{\partial x^2} + i\mu w(t, x) + iq |w(t, x)|^{2p} w(t, x)$$

from which we obtain the canonical form :

$$w(t, x) = w(0, x) + i\varepsilon \int_0^t \frac{\partial^2 w(z, x)}{\partial x^2} dz + i\mu \int_0^t w(z, x) dz + iq \int_0^t |w(z, x)|^{2p} w(z, x) dz.$$

Thus, we obtain the Adomian algorithm :

$$\begin{cases} w_0(t, x) = w(0, x) \\ w_{n+1}(t, x) = i\varepsilon \int_0^t \frac{\partial^2 w_n(z, x)}{\partial x^2} dz + i\mu \int_0^t w_n(z, x) dz + iq \int_0^t A_n(z, x) dz; n \geq 0 \end{cases}$$

Let's calculate the polynomials : A_0, A_1, A_2, \dots

$$\begin{cases} A_0 = |\beta|^{2p} w_0 \\ A_1 = w_1 (w_0 \overline{w_0})^p + p w_0 (w_1 \overline{w_0} + \overline{w_1} w_0)^{p-1} \\ A_2 = 2 (\overline{w_0} w_0)^p w_2 + 2p (\overline{w_0} w_1 + \overline{w_1} w_0)^{p-1} (w_1) + p(p-1) (2 (\overline{w_0} w_2 + w_1 \overline{w_1} + \overline{w_2} w_0))^{p-2} (w_0) \\ \dots \end{cases}$$

Let's calculate the terms : $w_0(t, x), w_1(t, x), w_2(t, x), \dots$

we thus obtain: .

$$\begin{cases} w_0(t, x) = \beta e^{iax} \\ w_1(t, x) = \beta i t (\mu - \varepsilon a^2 + q |\beta|^{2p}) e^{iax} \\ w_2(t, x) = \beta \frac{[i t (\mu - \varepsilon a^2 + q |\beta|^{2p})]^2}{2!} e^{iax} \\ w_3(t, x) = \beta \frac{[i t (\mu - \varepsilon a^2 + q |\beta|^{2p})]^3}{3!} e^{iax} \\ \dots \\ w_n(t, x) = \beta \frac{[i t (\mu - \varepsilon a^2 + q |\beta|^{2p})]^n}{n!} e^{iax} \end{cases}$$

Therefore, the solution of problem (E) obtained by the ADM method is : :

$$w(t, x) = \sum_{n=0}^{+\infty} w_n(t, x) = \beta \exp [i ((\mu - \varepsilon a^2 + q |\beta|^{2p}) t + ax)].$$

4 Resolution via Numerical Method SBA

Consider the following equation

$$(E) : \begin{cases} i \frac{\partial w(t, x)}{\partial t} + \varepsilon \frac{\partial^2 w(t, x)}{\partial x^2} + q |w(t, x)|^{2p} w(t, x) = 0 \\ w(0, x) = \beta e^{iax} \end{cases}$$

Let us determine the canonical form of Adomian, the equation

$$i \frac{\partial w(t, x)}{\partial t} + \varepsilon \frac{\partial^2 w(t, x)}{\partial x^2} + q |w(t, x)|^{2p} w(t, x) = 0 \tag{4.1}$$

is equivalent to

$$\frac{\partial w(t, x)}{\partial t} = i \varepsilon \frac{\partial^2 w(t, x)}{\partial x^2} + iq |w(t, x)|^{2p} w(t, x) \tag{4.2}$$

By putting

$$Nw(t, x) = iq |w(t, x)|^{2p} w(t, x)$$

from which we obtain the Adomian [7] canonical form:

$$w(t, x) = w(0, x) + i \varepsilon \int_0^t \frac{\partial^2 w(z, x)}{\partial x^2} dz + \int_0^t Nw(z, x) dz. \tag{4.3}$$

Applying to (4.3) the method of successive approximations [8], we obtain:

$$w^k(t, x) = w^k(0, x) + i \varepsilon \int_0^t \frac{\partial^2 w^k(z, x)}{\partial x^2} dz + \int_0^t Nw^{k-1}(z, x) dz, k \geq 1 \tag{4.4}$$

We thus obtain the SBA algorithm [9]:

$$\begin{cases} w_0^k(t, x) = w^k(0, x) + \int_0^t Nw^{k-1}(z, x) dz, k \geq 1 \\ w_{n+1}^k(t, x) = i\varepsilon \int_0^t \frac{\partial^2 w_n^k(z, x)}{\partial x^2} dz; n \geq 0 \end{cases} \quad (4.5)$$

Let's apply Picard's principle [10] to (4.5), at step $k = 1$, $Nw^0(t, x) = 0$, si $w^0(t, x) = 0$, hence

$$\begin{cases} w_0^1(t, x) = \beta e^{iax} + \int_0^t Nw^0(z, x) dz, k \geq 1 \\ w_{n+1}^1(t, x) = i\varepsilon \int_0^t \frac{\partial^2 w_n^1(z, x)}{\partial x^2} dz; n \geq 0 \end{cases}$$

Therefore we have

$$\begin{cases} w_0^1(t, x) = \beta e^{iax} \\ w_1^1(t, x) = -ia^2 \beta \varepsilon t e^{iax} \\ w_2^1(t, x) = -\frac{1}{2} a^4 t^2 \beta \varepsilon^2 e^{iax} \\ w_3^1(t, x) = \frac{1}{6} ia^6 t^3 \beta \varepsilon^3 e^{iax} \\ \dots \\ w_n^1(t, x) = \beta \frac{(-\varepsilon ia^2 t)^n}{n!} e^{iax}, n \geq 0 \end{cases}$$

from which at step $k = 1$, we obtain:

$$w^1(t, x) = \lim_{p \rightarrow +\infty} \beta e^{iax} \sum_{p=0}^n \frac{(-\varepsilon ia^2 t)^p}{p!} = \beta \exp [i(ax - \varepsilon a^2 t)].$$

Then let's calculate $Nw^1(t, x)$

$$Nw^1(t, x) = iq |w^1(t, x)|^{2p} w^1(t, x) - iq \beta^{2p} w^1(t, x) \neq 0$$

therefore, we modify problem (E) into an equivalent problem:

$$(E) : \begin{cases} i \frac{\partial w(t, x)}{\partial t} + \varepsilon \frac{\partial^2 w(t, x)}{\partial x^2} + q |\beta|^{2p} w(t, x) + \tilde{N}w(t, x) = 0 \\ w(0, x) = \beta e^{iax} \end{cases}$$

where

$$\tilde{N}w(t, x) = q |w(t, x)|^{2p} w(t, x) - q |\beta|^{2p} w(t, x)$$

Therefore, we obtain:

$$\frac{\partial w(t, x)}{\partial t} = \varepsilon i \frac{\partial^2 w(t, x)}{\partial x^2} + qi |\beta|^{2p} w(t, x) + i \tilde{N}w(t, x)$$

then the canonical form of Adomian [10]

$$w(t, x) = \beta e^{iax} + \varepsilon i \int_0^t \frac{\partial^2 w(z, x)}{\partial x^2} dz + qi |\beta|^{2p} \int_0^t w(z, x) dz + i \int_0^t \tilde{N}w(z, x) dz$$

The new algorithm is then:

$$\begin{cases} w_0^k(t, x) = \beta e^{iax} + i \int_0^t \tilde{N}w^{k-1}(z, x) dz; k \geq 1 \\ w_{n+1}^k(t, x) = \varepsilon i \int_0^t \frac{\partial^2 w_n^k(z, x)}{\partial x^2} dz + q |\beta|^{2p} i \int_0^t w_n^k(z, x) dz; n \geq 0 \end{cases}$$

Let's determine $w^1(t, x)$

$$\left\{ \begin{array}{l} w_0^1(t, x) = \beta e^{iax} \\ w_1^1(t, x) = \beta i ((q|\beta|^{2p} - a^2\varepsilon)t) e^{iax} \\ w_2^1(t, x) = \beta \frac{(i(q|\beta|^{2p} - a^2\varepsilon)t)^2}{2} e^{iax} \\ w_3^1(t, x) = \beta \frac{(i(q|\beta|^{2p} - a^2\varepsilon)t)^3}{3!} e^{iax} \\ \dots \\ w_n^1(t, x) = \beta \frac{(i(q|\beta|^{2p} - a^2\varepsilon)t)^n}{n!} e^{iax} \end{array} \right.$$

hence

$$w^1(t, x) = \beta e^{iax} \sum_{n=0}^{+\infty} \frac{(i(q|\beta|^{2p} - a^2\varepsilon)t)^n}{n!} = \beta \exp [i((q|\beta|^{2p} - a^2\varepsilon)t + ax)]$$

We thus obtain $\tilde{N}w^1(t, x) = 0$

Recursively we have:

$$w^1(z, x) = w^2(z, x) = \dots = w^k(z, x)$$

so the solution of problem (E) is:

$$w(t, x) = \beta \exp [i((q|\beta|^{2p} - a^2\varepsilon)t + ax)].$$

Consider the following problem:

$$(F) : \left\{ \begin{array}{l} i \frac{\partial w(t, x)}{\partial t} + \varepsilon \frac{\partial^2 w(t, x)}{\partial x^2} + \mu w(t, x) + q|w(t, x)|^{2p} w(t, x) = 0 \\ w(0, x) = \beta e^{iax} \end{array} \right.$$

We obtain the following Adomian algorithm:

$$w(t, x) = w(0, x) + i\varepsilon \int_0^t \frac{\partial^2 w(z, x)}{\partial x^2} dz + i\mu \int_0^t w(z, x) dz + iq \int_0^t Nw(z, x) dz \quad (4.6)$$

where

$$Nw(t, x) = |w(t, x)|^{2p} w(t, x)$$

Let us apply the method of successive approximations to (4.6),

$$\left\{ \begin{array}{l} w^k(t, x) = w^k(0, x) + i\varepsilon \int_0^t \frac{\partial^2 w^k(z, x)}{\partial x^2} dz + \\ i\mu \int_0^t w^k(z, x) dz + iq \int_0^t Nw^{k-1}(z, x) dz, k \geq 1 \end{array} \right. \quad (4.7)$$

We are looking for the solution of (F) in the form of a series [11]

$$w^k(t, x) = \sum_{n=0}^{+\infty} w_n^k(t, x)$$

At each step $k \geq 1$, we have the following algorithm [12]

$$\left\{ \begin{array}{l} w_0^k(t, x) = w^k(0, x) + iq \int_0^t Nw^{k-1}(z, x) dz \\ w_{n+1}^k(t, x) = i\varepsilon \int_0^t \frac{\partial^2 w_n^k(z, x)}{\partial x^2} dz + i\mu \int_0^t w_n^k(z, x) dz; n \geq 0 \end{array} \right.$$

Let's calculate the terms of the series

$$w^k(t, x) = \sum_{n=0}^{+\infty} w_n^k(t, x)$$

At step $k = 1$, for $w^0(t, x) = 0$, we have $Nw^0(t, x) = 0$ and we obtain :

$$\begin{cases} w_0^1(t, x) = \beta e^{iax} \\ w_1^1(t, x) = \beta i (\mu - a^2 \varepsilon) t e^{iax} \\ w_2^1(t, x) = \beta \frac{(i ((\mu - a^2 \varepsilon)) t)^2}{2!} e^{iax} \\ w_3^1(t, x) = \beta \frac{(i ((\mu - a^2 \varepsilon)) t)^3}{3!} e^{iax} \\ \dots \\ w_n^1(t, x) = \beta \frac{(i ((\mu - a^2 \varepsilon)) t)^n}{n!} e^{iax} \end{cases}$$

therefore

$$w^1(t, x) = \beta e^{iax} \sum_{n=0}^{+\infty} \frac{(i ((\mu - a^2 \varepsilon)) t)^n}{n!} = \beta \exp [i ((\mu - a^2 \varepsilon) t + ax)]$$

Calculate $N^1(t, x)$, we have :

$$Nw^1(t, x) = q |w^1(t, x)|^{2p} w^1(t, x) = q |\beta|^{2p} w^1(t, x) \neq 0$$

We then modify problem (F) into an equivalent problem:

$$(F) : \begin{cases} i \frac{\partial w(t, x)}{\partial t} + \varepsilon \frac{\partial^2 w(t, x)}{\partial x^2} + \mu w(t, x) + q |\beta|^{2p} w(t, x) + \tilde{N}w(z, x) = 0 \\ w(0, x) = \beta e^{iax} \end{cases} \quad (4.8)$$

where

$$\tilde{N}w(z, x) = q |w(t, x)|^{2p} w(t, x) - q |\beta|^{2p} w(t, x)$$

we have the following canonical form :

$$w(t, x) = w(0, x) + i \int_0^t \frac{\partial^2 w(z, x)}{\partial x^2} dz + i \mu \int_0^t w(z, x) dz + qi |\beta|^{2p} \int_0^t w(z, x) dz + i \int_0^t \tilde{N}w(z, x) dz$$

Let's apply the method of successive approximations to (4.7),

$$w^k(t, x) = w^k(0, x) + i\varepsilon \int_0^t \frac{\partial^2 w^k(z, x)}{\partial x^2} dz + i(\mu + q |\beta|^{2p}) \int_0^t w^k(z, x) dz + i \int_0^t \tilde{N}w^{k-1}(z, x) dz, k \geq 1$$

Thus, at each step $k \geq 1$, the following algorithm is obtained:

$$\begin{cases} w_0^k(t, x) = w^k(0, x) + i \int_0^t \tilde{N}w^{k-1}(z, x) dz, k \geq 1 \\ w_{n+1}^k(t, x) = i\varepsilon \int_0^t \frac{\partial^2 w_n^k(z, x)}{\partial x^2} dz + i(\mu + q |\beta|^{2p}) \int_0^t w_n^k(z, x) dz; n \geq 0 \end{cases}$$

Let's calculate $w^1(t, x)$ at step $k = 1$

$$\begin{cases} w_0^1(t, x) = w^1(0, x) + i \int_0^t \tilde{N}w^0(z, x) dz \\ w_{n+1}^1(t, x) = i\varepsilon \int_0^t \frac{\partial^2 w_n^1(z, x)}{\partial x^2} dz + i(\mu + q |\beta|^{2p}) \int_0^t w_n^1(z, x) dz; n \geq 0 \end{cases}$$

for $w^0(t, x) = 0$ we have : $Nw^0(t, x) = 0$, hence

$$\begin{cases} w_0^1(t, x) = w^k(0, x) \\ w_{n+1}^1(t, x) = i\varepsilon \int_0^t \frac{\partial^2 w_n^1(z, x)}{\partial x^2} dz + i\mu \int_0^t w_n^1(z, x) dz + qi|\beta|^{2p} \int_0^t w_n^1(z, x) dz; n \geq 0 \end{cases}$$

$$\begin{cases} w_0^1(t, x) = \beta e^{iax} \\ w_1^1(t, x) = \beta it (\mu - a^2\varepsilon + q|\beta|^{2p}) e^{iax} \\ w_2^1(t, x) = \beta \frac{(it (\mu - a^2\varepsilon + q|\beta|^{2p}))^2}{2!} e^{iax} \\ w_3^1(t, x) = \beta \frac{(it (\mu - a^2\varepsilon + q|\beta|^{2p}))^3}{3!} e^{iax} \\ \dots \\ w_n^1(t, x) = \beta \frac{(it (\mu - a^2\varepsilon + q|\beta|^{2p}))^n}{n!} e^{iax} \end{cases}$$

the solution at step $k = 1$ is

$$\begin{aligned} w^1(t, x) &= \beta e^{iax} \sum_{n=0}^{+\infty} \frac{(it (\mu - a^2\varepsilon + q|\beta|^{2p}))^n}{n!} \\ &= \beta \exp [i ((\mu - a^2\varepsilon + q|\beta|^{2p}) t + ax)] \end{aligned}$$

We thus obtain:

$$w^1(t, x) = w^2(t, x) = \dots = w^k(t, x) = \beta \exp [i ((\mu - a^2\varepsilon + q|\beta|^{2p}) t + ax)].$$

Thus the solution of problem (F) :

$$\begin{aligned} w(t, x) &= \lim_{k \rightarrow +\infty} w^k(t, x) \\ &= \beta \exp [i ((\mu - a^2\varepsilon + q|\beta|^{2p}) t + ax)] \end{aligned}$$

5 Conclusions

The SBA and ADM methods have allowed us to successfully solve the Ivancevic option pricing model (IOPM) in financial mathematics, the Schrödinger model in quantum mechanics and the classical Black-Scholes equation.

Competing Interests

Authors have declared that no competing interests exist.

References

- [1] Pare Daouda, Kassienou Lamien, et al. Solving the Ivancevic options pricing model with the numerical method SBA. *Advances in Differential Equations and Control Processes*. 2021;14(2):133-143.
- [2] Gonzalez-Gaxiola O, Edeki SO, et al. Solving the Ivancevic Pricing Model Using the He's Frequency Amplitude Formulation. *EJPAM*. 2017;10(4):631-637.
- [3] Chengri Jin, Mingzhu. A new modification of Adomian decomposition method for solving a kind of evolution. *Appl. Math. Comput*. 2005;169.

- [4] Abbaoui K, Cherruault Y. The decomposition method applied to the Cauchy problem. *Kybernetes*. 1999;28(1):68-74.
- [5] Adomian G. *Nonlinear stochastic systems theory and application to physics*. Kluwe Academic Publishers; 1989.
- [6] Abbaoui K, Cherruault Y. Convergence of Adomian method applied to differential equations. *Math. Comput. Modelling*. 1994;28(5):103-109.
- [7] Bakari Abbo. *Nouvel algorithme numérique de résolution des Equations Différentielles Ordinaires (EDO) et des Equations aux Dérivées Partielles (EDP) non linéaires*. Thèse de Doctorat Unique, Université de Ouagadougou (Burkina Faso), Janvier; 2007.
- [8] Gires Dimitri N'Kaya, Justin Mouyedo Loufouilou, et al. SBA Method in solving the Schrödinger equation. *International. J. Func. Analys, Oper. Theorie and Appli*. 2018;10(1):1-10.
- [9] Yaro R. *Contribution à la résolution de quelques modèles mathématiques de la dynamique des populations par les méthodes d'Adomian, SBA et des perturbations*. Thèse de Doctorat Unique, Université de Ouagadougou; 2015.
- [10] Yaya Moussa, Youssouf Pare, et al. New approach of the Adomian decomposition method. *Inter. J. Num. Methods. Appl*. 2017;16(1):1-17.
- [11] Youssouf Pare. *Résolution de quelques équations fonctionnelles par la méthode SBA(Somé Blaise-Abbo)*. Thèse de Doctorat Unique, Université de Ouagadougou; 2010.
- [12] Youssouf Pare. *Equations intégrales et intégrro-différentielles*. Generis Publishing; 2021.

© 2022 Adama et al.; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:

The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)

<https://www.sdiarticle5.com/review-history/84786>