



# Application of Alpha Power Cubic Transmuted Dagum Distribution to Real-Life Data

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## Authors' contributions

*This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.*

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## ABSTRACT

In this article, a new model is proposed called alpha power cubic transmuted distribution. Some properties of the new distribution such as survival and hazard functions and other useful measures were obtained. The model parameters were estimated using the method of maximum likelihood. The usefulness of the proposed distribution has been shown using three real datasets. This indicates that the proposed compound distribution would be useful in the area of distribution theory in Statistics, and in modeling real life data sets relating to biology, demography, geology, economics, environmental sciences, engineering, finance, medicine, hydrology, insurance and many other fields.

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## 1. INTRODUCTION

The Dagum distribution was proposed by [1]. His proposals brought about the improvement of statistical distributions for fitting empirical income and wealth data, that could be adequate for both heavy tails in empirical income and wealth distributions, and also allow for interior mode. Dagum distribution has two types; the Type-I involves three parameter and Type-II deals with four parameters. Dagum in 1977 motivated his model from empirical observation that the income elasticity of the cumulative distribution function (cdf) of income into a decreasing and bounded function of F.

Recently, very useful families of probability distributions have been proposed in the literature and it has been shown that they are useful for adding skewness and flexibility to other models. A brief summary of the recently proposed families include the a new generalized Weibull-G family by [2], Logistic-X family by [3], a new Weibull-G family by [4], a Lindley-G family by [5], a Gompertz-G family by [6], an odd Lindley-G family by [7], Alpha Power transformation by [8], odd Lomax generator of distributions (Odd Lomax-G family) by [9] and Poisson-X family by [10] as so on.

A good research into these families of distributions has led to the proposition of some new distributions which include; the exponential-Lindley distribution by [11], transmuted normal distribution by [12], alpha power Weibull distribution by [13], odd Lindley-Rayleigh distribution by [14], the transmuted odd Lindley-Rayleigh distribution by [15], alpha power generalized exponential distribution by [16], odd Lindley-inverse exponential distribution by [17] and cubic transmuted Dagum distribution by [18], etc.

According to [18], “the cubic transmuted Dagum distribution based on the cubic transmuted family was judged to be better than the transmuted Dagum distribution and the conventional Dagum distribution after its application to real life data sets”.

Following the above assessment of the cubic transmuted Dagum distribution, this article will develop an extension of the distribution (cubic transmuted Dagum distribution) using the alpha power transformation proposed by [8] with the

assertion that it will give better result when applied to real life datasets.

From [18], the cumulative distribution function (cdf) and probability density function (pdf) of the cubic transmuted Dagum distribution are given by:

$$F(x) = (1-\lambda)(1+\gamma x^{-\theta})^{-\beta} + 3\lambda(1+\gamma x^{-\theta})^{-2\beta} - 2\lambda(1+\gamma x^{-\theta})^{-3\beta} \quad (1)$$

and

$$f(x) = \gamma\theta\beta x^{-\theta-1}(1+\gamma x^{-\theta})^{-\beta-1} [1-\lambda+6\lambda(1+\gamma x^{-\theta})^{-\beta} - 6\lambda(1+\gamma x^{-\theta})^{-2\beta}] \quad (2)$$

respectively.

Where,  $x > 0, \gamma > 0, \theta > 0, \beta > 0, -1 \leq \lambda \leq 1$  and  $\gamma$  is a scale parameter while  $\theta$  and  $\beta$  are the shape parameters and  $\lambda$  is the transmuted parameter of the cubic transmuted Dagum distribution (CTDaD).

The remaining contents of this article are given in sections, the proposed distribution is defined in section two, the properties of the proposed distribution are presented in section three and the maximum likelihood approach to estimating its parameters is outlined in section four. An application of the proposed distribution to real-life datasets is provided in section five and a summary and conclusion is presented in section six.

## 2. METHODS AND MATERIALS

### 2.1 Definition of Apha Power Cubic Transmuted Dagum distribution (APCTDaD)

According to [8], the cumulative distribution function (cdf) and the probability density function (pdf) of the Alpha Power transformed family of distributions are defined as:

$$F(x) = \frac{\alpha^{G(x)} - 1}{\alpha - 1} \quad (3)$$

and

$$f(x) = \frac{\log(\alpha)}{(\alpha - 1)} g(x) \alpha^{G(x)} \quad (4)$$

“respectively, where  $g(x)$  and  $G(x)$  stand for the pdf and cdf of the cubic transmuted Dagum distribution or any continuous distribution to be extended respectively while  $\alpha > 0$  ( $\alpha \neq 1$ ) is the power/shape parameter of the alpha power

family which helps induce additional skewness and flexibility in the modified model.

Substituting equation (1) and (2) into equation (3) and (4) above and simplifying, the cdf and pdf of APCTDaD are obtained respectively as:

$$F(x) = \frac{\alpha \left( (1-\lambda)(1+\gamma x^{-\theta})^{-\beta} + 3\lambda(1+\gamma x^{-\theta})^{-2\beta} - 2\lambda(1+\gamma x^{-\theta})^{-3\beta} \right) - 1}{\alpha - 1} \tag{5}$$

and

$$f(x) = \frac{\log(\alpha) \gamma \theta \beta x^{-\theta-1} (1+\gamma x^{-\theta})^{-\beta-1} \left[ 1 - \lambda + 6\lambda(1+\gamma x^{-\theta})^{-\beta} - 6\lambda(1+\gamma x^{-\theta})^{-2\beta} \right]}{(\alpha - 1) \alpha^{- (1-\lambda)(1+\gamma x^{-\theta})^{-\beta} + 3\lambda(1+\gamma x^{-\theta})^{-2\beta} - 2\lambda(1+\gamma x^{-\theta})^{-3\beta}}} \tag{6}$$

Where,  $x, \alpha, \theta, \gamma, \beta > 0, -1 \leq \lambda \leq 1$  and  $\alpha \neq 1$  is a power parameter and  $\gamma$  is a scale parameter while  $\theta$  and  $\beta$  are the shape parameters and  $\lambda$  is the cubic transmuted parameter of the cubic transmuted Dagum distribution (CTDaD).

### 2.3 Graphical Presentation of Pdf and Cdf of APCTDaD

The plots pdf and cdf of the APCTDaD using some parameter values are displayed in Fig. 1 and Fig. 2 as follows.

Considering the plot of the pdf of the ATCTDaD in Fig. 1, it is clear that the distribution is skewed and flexible and that its shape depend on the values of the parameters. It is also clear that the plots of the pdf and cdf are in line with the limiting behavior of a probability distribution.

### 2.4 Reliability Analysis of the APCTDaD.

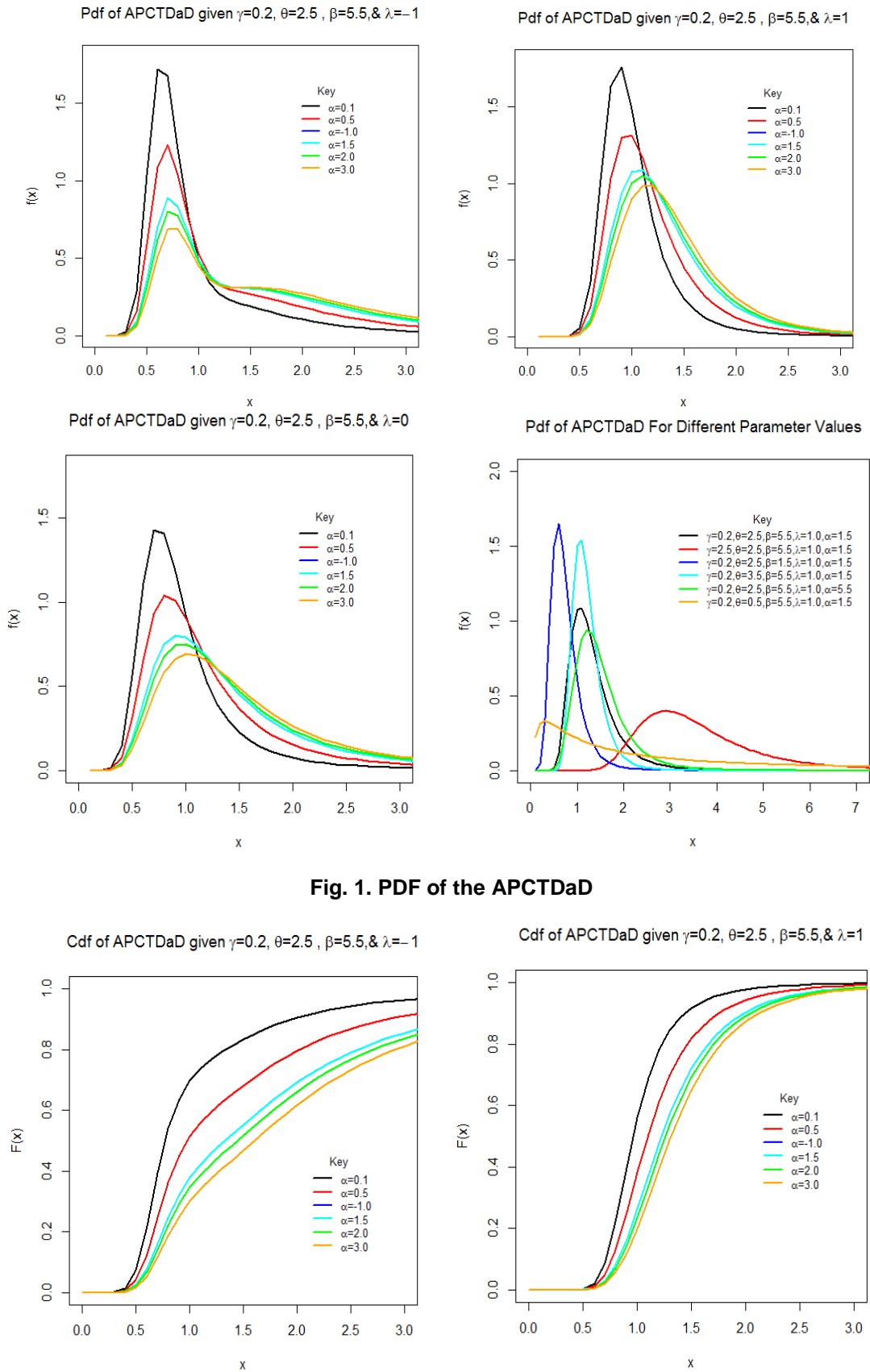
A Survival function states the chances that a system will perform its designed purpose within a specified time limit. It is defined by the following equation:

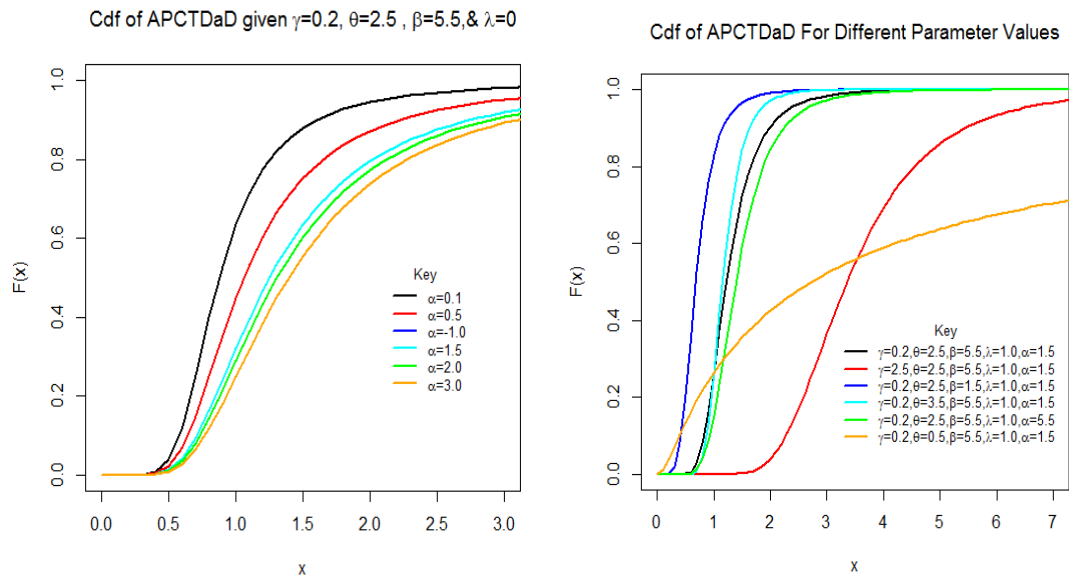
$$S(x) = 1 - F(x) \tag{7}$$

Utilizing the cdf of the proposed APCTDaD in (7), the survival function for APCTDaD is obtained as:

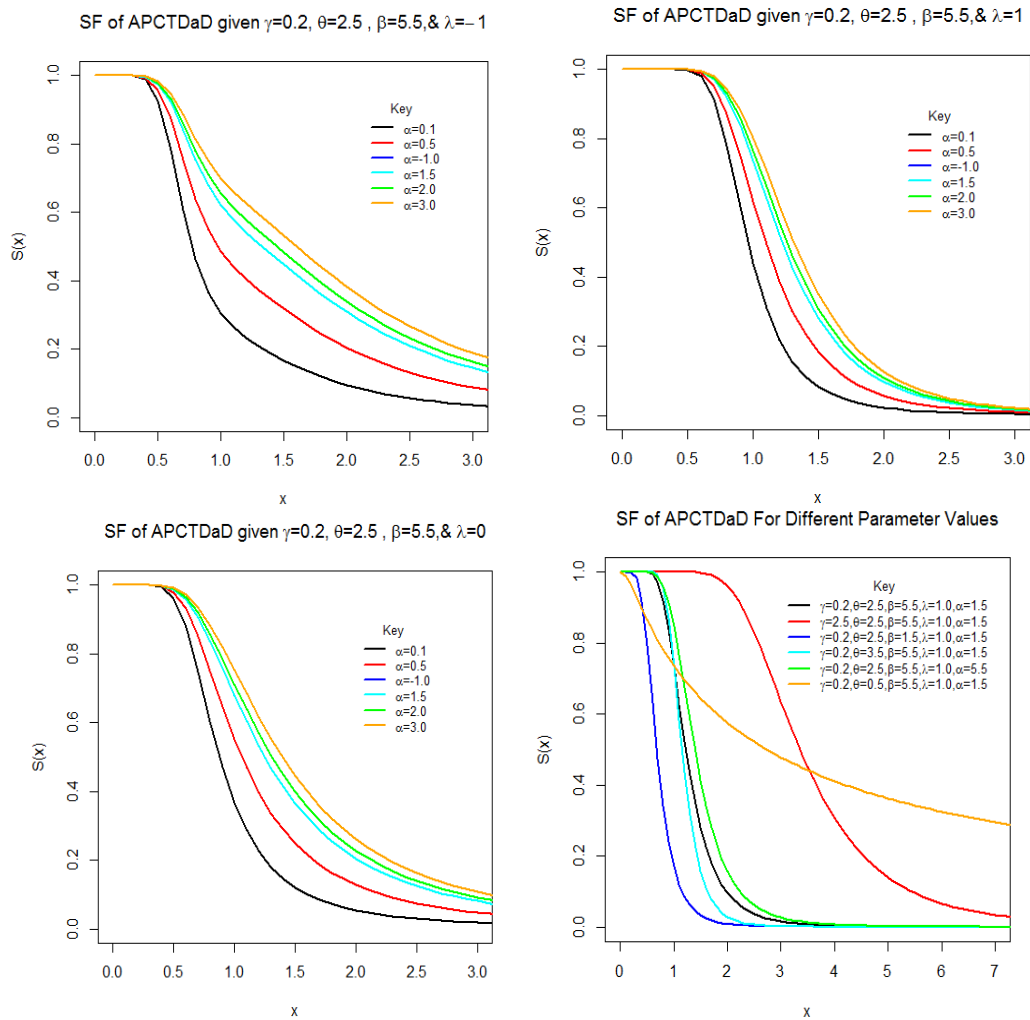
$$S(x) = 1 - \frac{\alpha \left( (1-\lambda)(1+\gamma x^{-\theta})^{-\beta} + 3\lambda(1+\gamma x^{-\theta})^{-2\beta} - 2\lambda(1+\gamma x^{-\theta})^{-3\beta} \right) - 1}{\alpha - 1} \tag{8}$$

A plot of the function expressed in equation (8) is given in Fig. 3.





**Fig. 2. CDF of the APCTDaD**



**Fig. 3. Survival function of APCTDaD**

Based on the plots of the survival function in Fig. 3 above, it can be seen that the function is decreasing, that is probability of survival decreases over time or survival rate decreases with increase in age or time.

within an interval of time. It is defined mathematically as:

$$h(x) = \frac{f(x)}{S(x)} = \frac{f(x)}{1-F(x)} \tag{9}$$

Hazard function: The hazard function describes the likelihood that a unit or component will fail

Hence, the hazard rate for the APCTDaD is given by:

$$h(x) = \frac{\log(\alpha)\gamma\theta\beta x^{-\theta-1}(1+\gamma x^{-\theta})^{-\beta-1} \left[ 1 - \lambda + 6\lambda(1+\gamma x^{-\theta})^{-\beta} - 6\lambda(1+\gamma x^{-\theta})^{-2\beta} \right]}{(\alpha-1)\alpha^{-\left((1-\lambda)(1+\gamma x^{-\theta})^{-\beta} + 3\lambda(1+\gamma x^{-\theta})^{-2\beta} - 2\lambda(1+\gamma x^{-\theta})^{-3\beta}\right)}} \tag{10}$$

$$1 - \frac{\alpha^{\left((1-\lambda)(1+\gamma x^{-\theta})^{-\beta} + 3\lambda(1+\gamma x^{-\theta})^{-2\beta} - 2\lambda(1+\gamma x^{-\theta})^{-3\beta}\right)} - 1}{\alpha - 1}$$

The following is a plot of the hazard function for some chosen parameter values

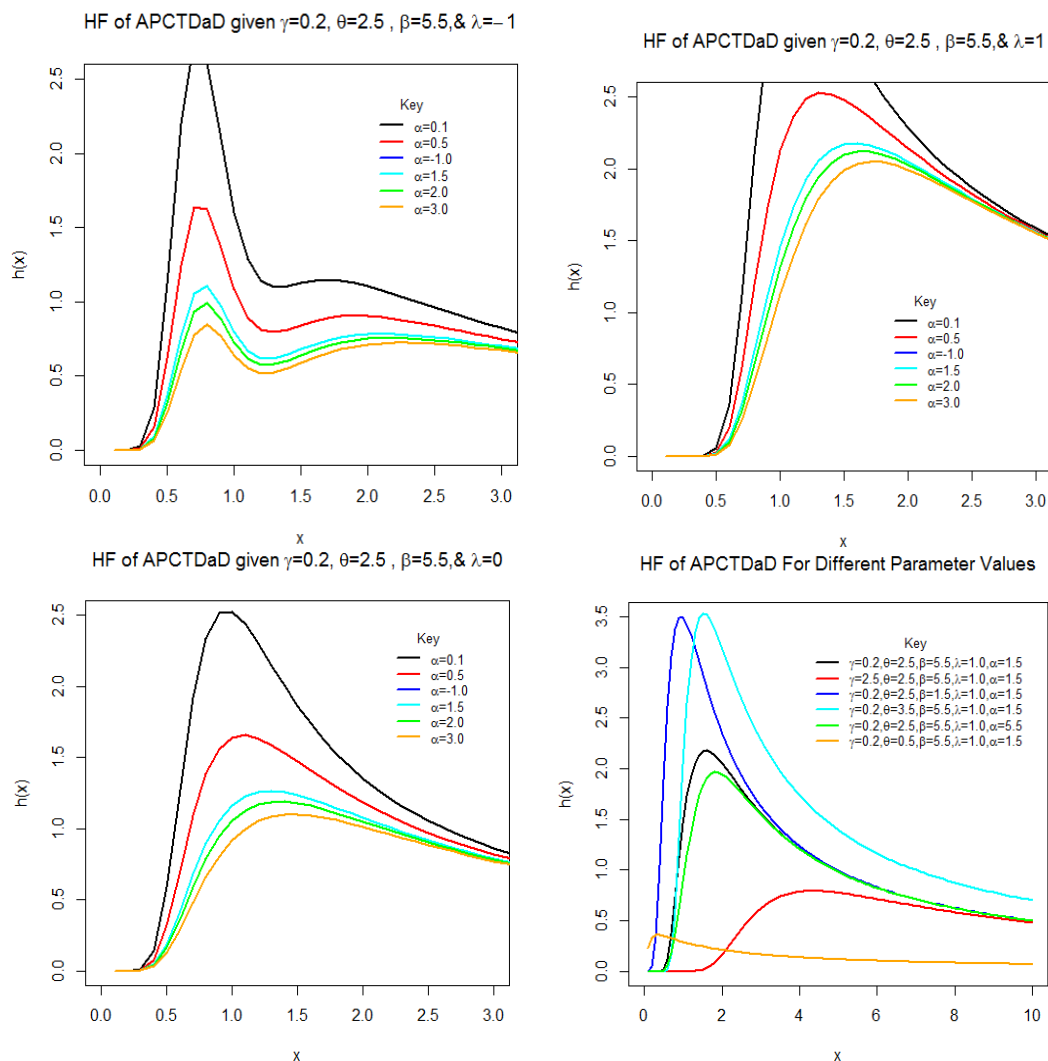


Fig. 4. Hazard function of APCTDaD

From Fig. 4 it can be seen that the plots of the hazard rate of the distribution are upside down bathtub hazard rate, meaning that the proposed model would be appropriate for analyzing random variables or events whose failure rate could be increasing and decreasing throughout the entire process.

### 2.5. Estimation of unknown Parameters of the APCTDaD

Let  $X_1, X_2, \dots, X_n$  be a sample of size "n" independently and identically distributed random variables from the APCTDaD with unknown parameters  $\alpha, \theta, \gamma, \lambda$  and  $\theta$  defined previously.

The likelihood function is given by:

$$L(X | \alpha, \beta, \theta, \gamma, \lambda) = \frac{(\log(\alpha)\gamma\theta\beta)^n \prod_{i=1}^n \left( x_i^{-\theta-1} (1+\gamma x_i^{-\theta})^{-\beta-1} \left[ 1 - \lambda + 6\lambda(1+\gamma x_i^{-\theta})^{-\beta} - 6\lambda(1+\gamma x_i^{-\theta})^{-2\beta} \right] \right)}{(\alpha-1)^n \prod_{i=1}^n \left( \alpha^{-(-1-\lambda)(1+\gamma x_i^{-\theta})^{-\beta} + 3\lambda(1+\gamma x_i^{-\theta})^{-2\beta} - 2\lambda(1+\gamma x_i^{-\theta})^{-3\beta}} \right)} \tag{11}$$

Taking the partial derivative of the natural logarithm of the likelihood function equated to zero with respect to the parameters and solving for the solution of the non-linear system of equations produce the maximum likelihood estimates of parameters  $\alpha, \beta, \lambda, \theta$  and  $\gamma$ . Understand that it is not easy to solve the equations analytically and hence Newton-Raphson's iteration method is used with available data.

### 3. RESULTS AND DISCUSSION

In this section, three applications of the proposed model to real life datasets are provided to illustrate the flexibility of the APCTDaD introduced in Section 2. The MLEs of the model parameters are determined and some goodness-of-fit statistics for this distribution are compared with other competitive models. For all the datasets, the fits of APCTDaD is compared with those of the cubic transmuted Dagum distribution (CTDaD), transmuted Dagum distribution (TDaD) and the conventional Dagum distribution (DaD).

It is important to note that after application of the distributions to the data, the selection of the fitted probability models is based on some information criteria. These information criteria are in relation to the value of the log-likelihood function evaluated at the maximum likelihood estimates ( $\ell$ ) and include: Akaike Information Criterion

(AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC) and Hannan Quin Information Criterion (HQIC). Other model selection measures used in this research are: Anderson-Darling (A\*), Cramèr-Von Mises (W\*) and Kolmogorov-smirnov (K-S) statistics. For a comprehensive study of these measures the readers can check [19]. Meanwhile, the smaller the values of these statistics are for a particular model, the better the fit of the distribution model. The necessary computations in this research were carried out using the package "AdequacyModel" in R software which is freely available online from <http://cran.r-project.org/web/packages/AdequacyModel/AdequacyModel.pdf>.

#### 3.1 Application 1: Based on Dataset I

This data represents the survival times of a group of patients suffering from head and neck cancer diseases and treated using a combination of radiotherapy and chemotherapy (RT+CT) [20-22].

The following table and figures present a good exploration of the dataset with some explanations.

Considering the dataset in Table 1 and Fig. 5, it can be seen that data set I is skewed to the right or positively skewed and hence suitable for skewed models like the proposed APCTDaD.

**Table 1. Descriptive Statistics for dataset I**

n	Minimum	$Q_1$	Median	$Q_3$	Mean	Maximum	Variance	Skewnes	Kurtosis
44	12.20	67.21	128.5	219.0	223.48	1776.00	93286.4	3.38382	13.5596

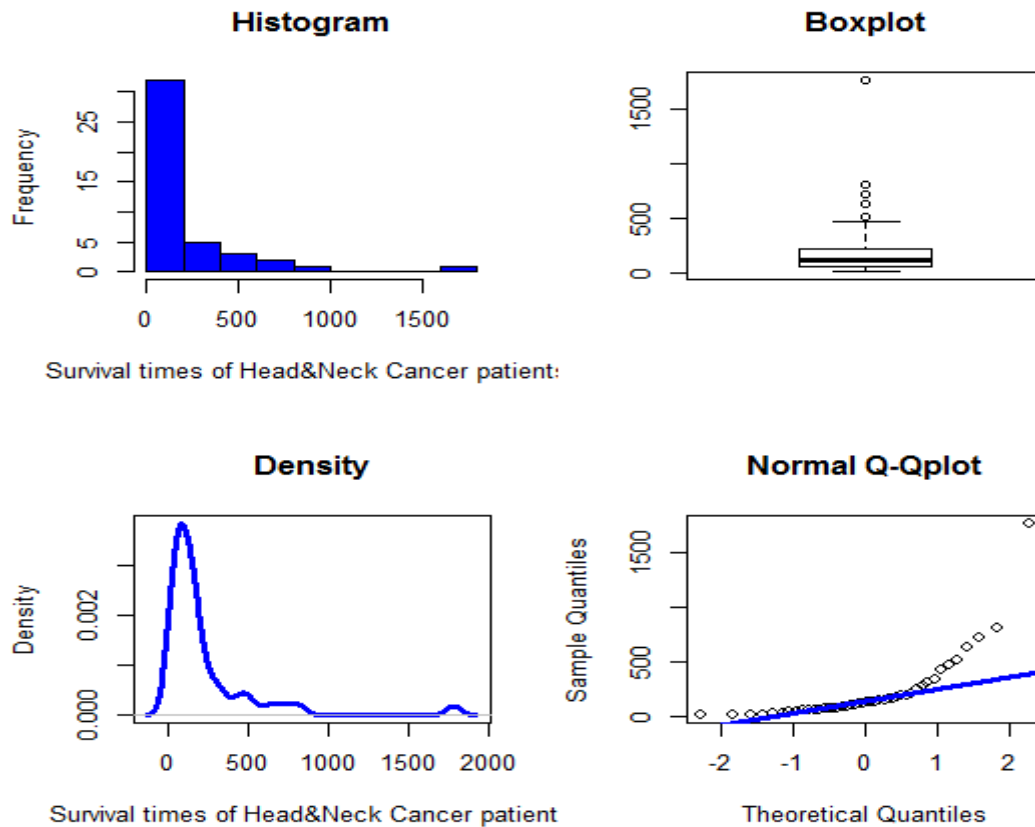


Fig. 5. A graphical summary of the dataset I

Table 2. Maximum Likelihood Parameter Estimates for dataset I

Distribution	$\hat{\gamma}$	$\hat{\theta}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\alpha}$
APCTDaD	2.6115052	0.8723578	9.9574230	0.9234020	8.1805877
CTDaD	9.0478552	0.8359463	4.4704835	0.8879190	-
TDaD	9.3003812	0.8012132	6.9245910	0.9839051	-
DaD	8.4264614	0.9986907	8.9957360	-	-

Table 3. The statistics  $\ell$ , AIC, CAIC, BIC and HQIC for dataset I

Distribution	$\hat{\ell}$	AIC	CAIC	BIC	HQIC	Ranks
APCTDaD	277.9149	563.8297	564.8554	570.9665	566.4764	1 <sup>st</sup>
CTDaD	277.8219	563.6439	564.6695	570.7806	566.2905	2 <sup>nd</sup>
TDaD	277.8373	565.6745	567.2535	574.5955	568.9828	3 <sup>rd</sup>
DaD	279.5193	565.0386	565.6386	570.3912	567.0236	4 <sup>th</sup>

Table 4. The  $A^*$ ,  $W^*$ , K-S statistic and P-values for dataset I

Distribution	$A^*$	$W^*$	K-S	P-Value (K-S)	Ranks
APCTDaD	0.1708722	0.02519533	0.063566	0.9893	1 <sup>st</sup>
CTDaD	0.1479777	0.02205189	0.076078	0.944	2 <sup>nd</sup>
TDaD	0.1489243	0.02100251	0.08537	0.8785	3 <sup>rd</sup>
DaD	0.3116772	0.04943709	0.10655	0.6606	4 <sup>th</sup>



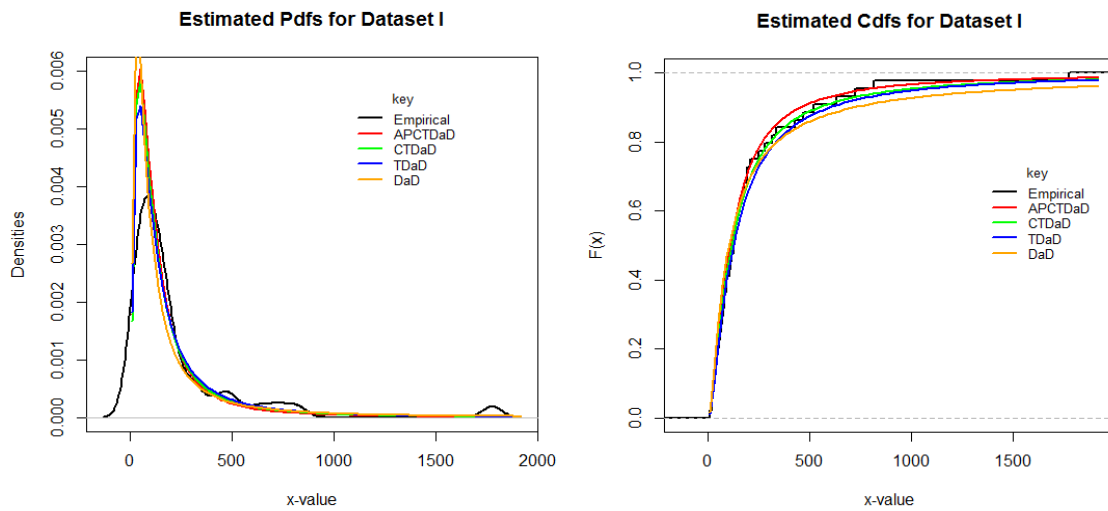


Fig. 6. Plots of the estimated densities and cdfs of the fitted distributions to dataset I

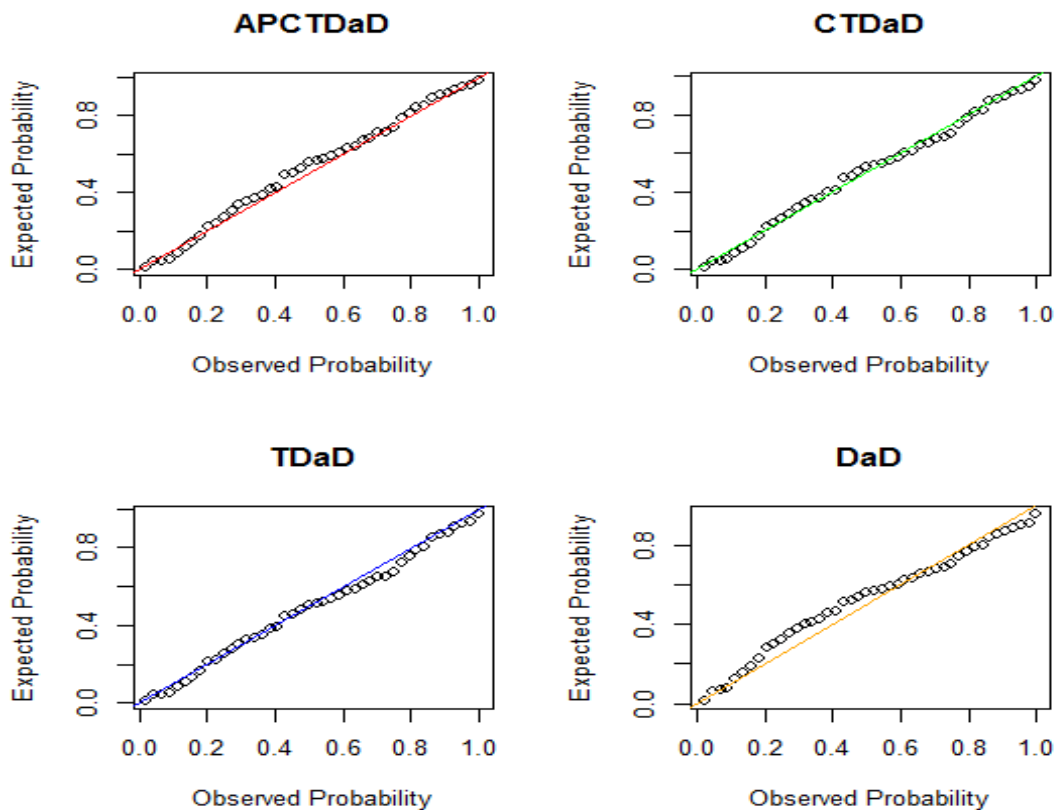


Fig. 7. Probability plots for the fit of the APCTDaD, CTDaD, TDaD & DaD based on dataset I

### 3.2 Application 2: Based on Dataset II

Dataset II is on the remission times of a random sample of 128 bladder cancer patients in months. This particular data has been used previously by

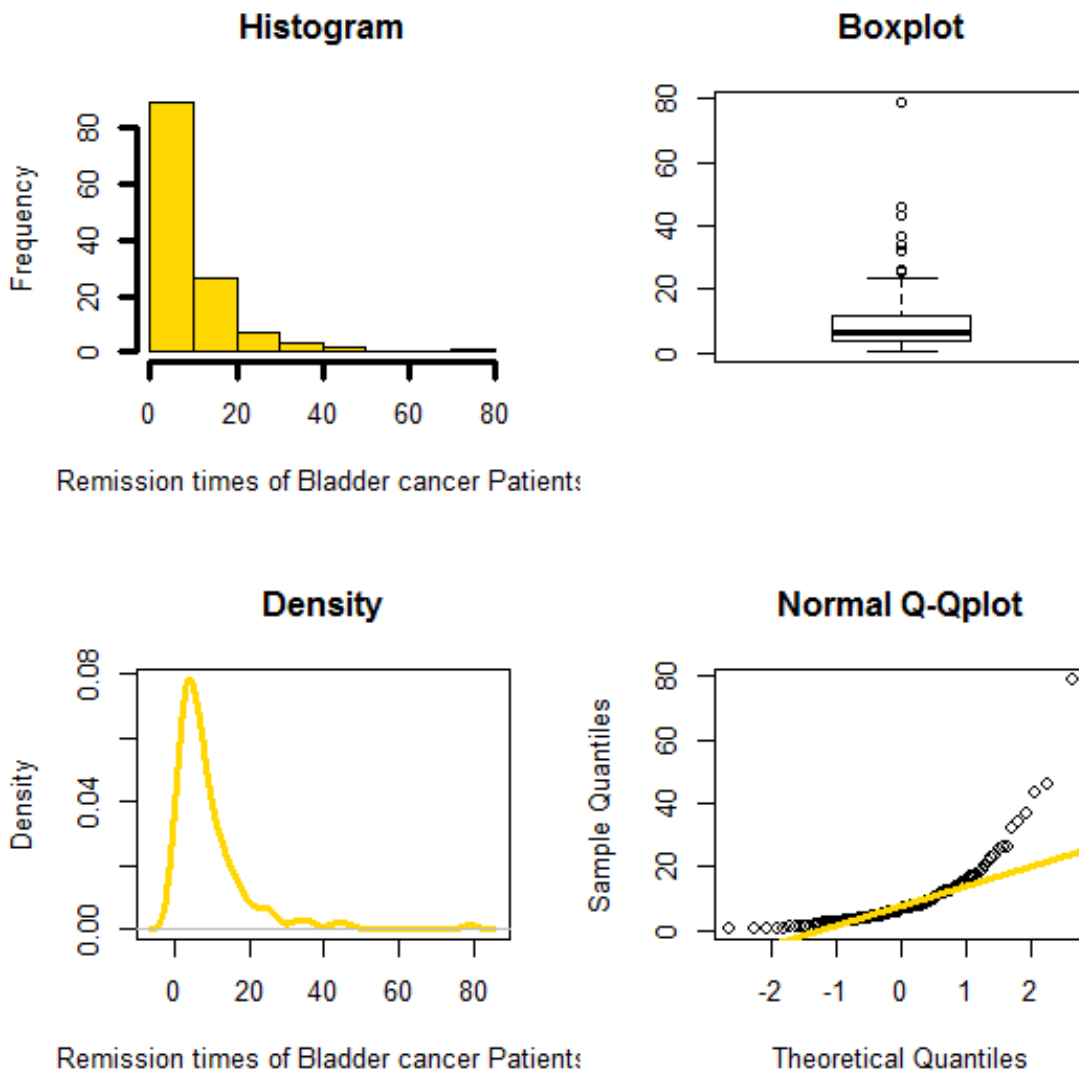
[23-27]. Its values and summary statistics is as follows:

0.080, 0.200, 0.400, 0.500, 0.510, 0.810, 0.900, 1.050, 1.190, 1.260, 1.350, 1.400, 1.460, 1.760,

2.020, 2.020, 2.070, 2.090, 2.230, 2.260, 2.460, 7.660, 7.870, 7.930, 8.260, 8.370, 8.530, 8.650,  
 2.540, 2.620, 2.640, 2.690, 2.690, 2.750, 2.830, 8.660, 9.020, 9.220, 9.470, 9.740, 10.06, 10.34,  
 2.870, 3.020, 3.250, 3.310, 3.360, 3.360, 3.480, 10.66, 10.75, 11.25, 11.64, 11.79, 11.98, 12.02,  
 3.520, 3.570, 3.640, 3.700, 3.820, 3.880, 4.180, 12.03, 12.07, 12.63, 13.11, 13.29, 13.80, 14.24,  
 4.230, 4.260, 4.330, 4.340, 4.400, 4.500, 4.510, 14.76, 14.77, 14.83, 15.96, 16.62, 17.12, 17.14,  
 4.870, 4.980, 5.060, 5.090, 5.170, 5.320, 5.320, 17.36, 18.10, 19.13, 20.28, 21.73, 22.69, 23.63,  
 5.340, 5.410, 5.410, 5.490, 5.620, 5.710, 5.850, 25.74, 25.82, 26.31, 32.15, 34.26, 36.66, 43.01,  
 6.250, 6.540, 6.760, 6.930, 6.940, 6.970, 7.090, 46.12, 79.05.  
 7.260, 7.280, 7.320, 7.390, 7.590, 7.620, 7.630,

**Table 5. Summary statistics for the data set II**

parameters	n	Minimum	$Q_1$	Median	$Q_3$	Mean	Maximum	Variance	Skewness	Kurtosis
Values	128	0.0800	3.348	6.395	11.840	9.366	79.05	110.425	3.3257	19.1537



**Fig. 8. A graphical summary of the dataset II**

Also, looking at the summary statistics and graphical presentation of dataset II in Table 5 and Fig. 8 respectively, it is still clear that data set II is skewed to the right or positively skewed and hence good for skewed models like the proposed APCTDaD.

4.3, Study M.: 10, 33, 44,56, 59, 72, 74, 77, 92, 93, 96, 100, 100, 102, 105, 107, 107, 108, 108, 108, 109, 112, 113, 115, 116,120, 121, 122, 122, 124, 130, 134, 136, 139, 144, 146, 153, 159, 160, 163, 163, 168, 171, 172, 176,183, 195, 196, 197, 202, 213, 215, 216, 222, 230,231, 240, 245, 251, 253, 254, 255, 278, 293, 327,342, 347, 361, 402, 432, 458, 555.

### 3.3 Application 3: Based on Dataset III

Dataset III is on the survival times of 72 guinea pigs infected with virulenttubercle bacilli in days used previously by [27]. They are the Regiment

The following table and figures present a good exploration of the dataset with some explanations.

**Table 6. Maximum Likelihood Parameter Estimates for dataset II**

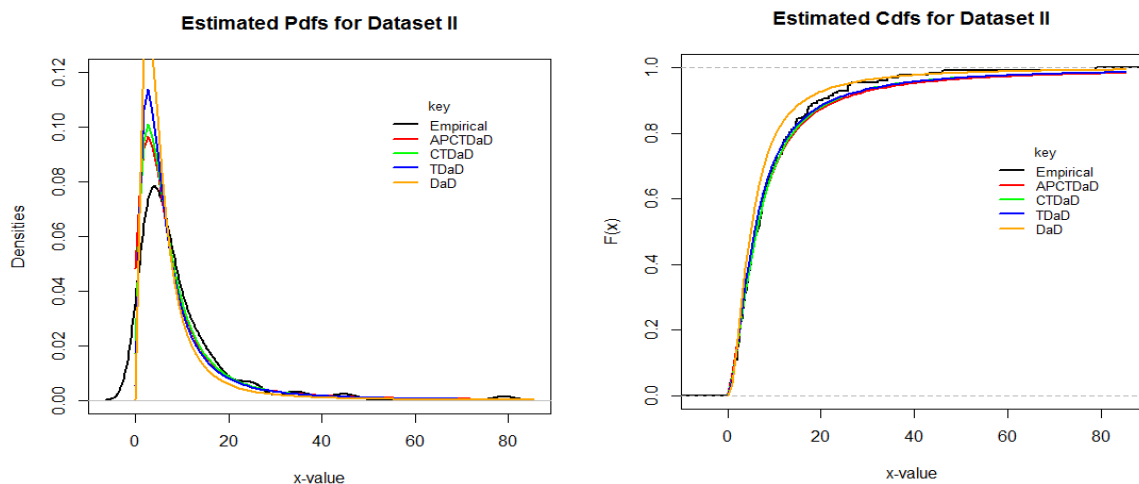
Distribution	$\hat{\gamma}$	$\hat{\theta}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\alpha}$
APCTDaD	9.2007268	1.3511256	0.6522685	0.5313419	7.7493862
CTDaD	9.8212178	1.2807240	1.0175680	0.6221916	-
TDaD	6.1177948	1.5123881	1.1742541	-0.6486219	-
DaD	8.235246	1.712200	1.574800	-	-

**Table 7. The statistics  $\ell$ , AIC, CAIC, BIC and HQIC for dataset II**

Distribution	$\hat{\ell}$	AIC	CAIC	BIC	HQIC	Ranks
APCTDaD	411.3755	830.7511	831.0763	842.1592	835.3863	1 <sup>st</sup>
CTDaD	411.5118	833.0236	833.5154	847.2838	838.8176	2 <sup>nd</sup>
TDaD	413.2724	834.5448	834.87	845.9529	839.18	3 <sup>rd</sup>
DaD	420.3174	846.6349	846.8284	855.191	850.1113	4 <sup>th</sup>

**Table 8. The  $A^*$ ,  $W^*$ , K-S statistic and P-values for dataset II**

Distribution	$A^*$	$W^*$	K-S	P-Value (K-S)	Ranks
APCTDaD	0.2838934	0.03962499	0.040366	0.9852	1 <sup>st</sup>
CTDaD	0.2021853	0.02865787	0.040712	0.9838	2 <sup>nd</sup>
TDaD	0.6238288	0.09281483	0.069858	0.5599	3 <sup>rd</sup>
DaD	1.088383	0.1649073	0.14645	0.008252	4 <sup>th</sup>



**Fig. 9. Plots of the estimated densities and cdfs of the fitted distributions to dataset II**

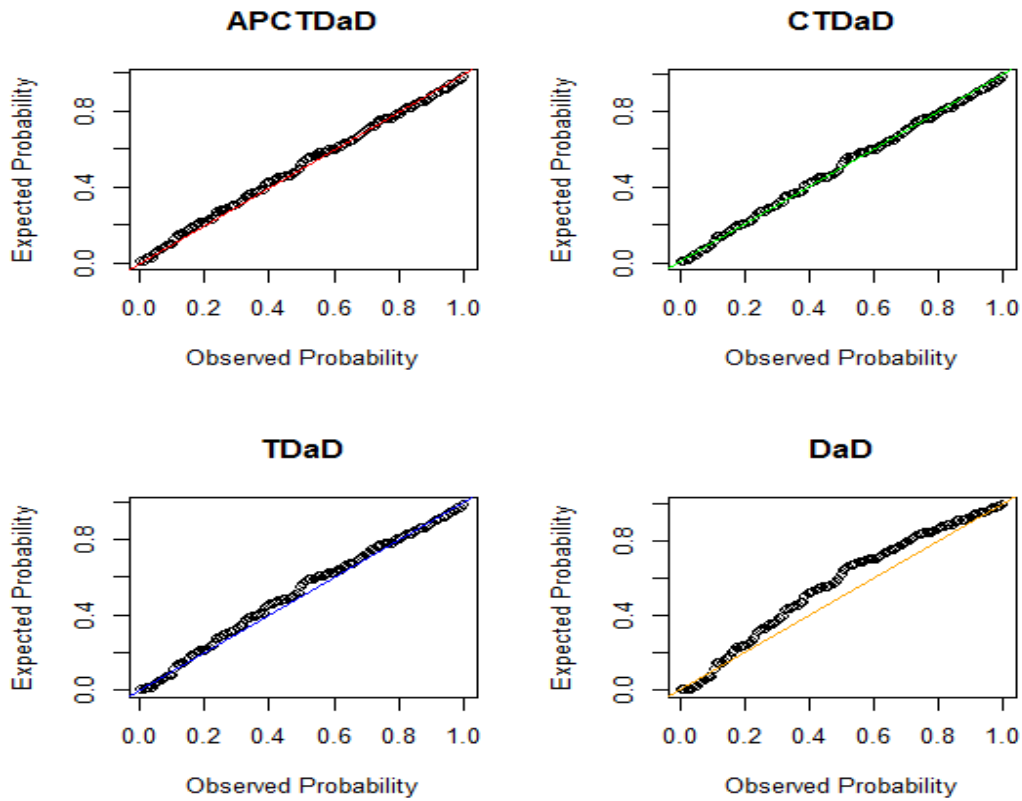


Fig. 10. Probability plots for the fit of the APCTDaD, CTDaD, TDaD & DaD based on dataset II

Table 9. Descriptive statistics for dataset III

Parameters	n	Minimum	$Q_1$	Median	$Q_3$	Mean	Maximum	Variance	Skewness	Kurtosis
Dataset A	72	10.0	108.0	149.5	224.0	176.8	555.0	10705.1	1.34128	1.98852

Table 10. Maximum likelihood parameter estimates for dataset III

Distribution	$\hat{\gamma}$	$\hat{\theta}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\alpha}$
APCTDaD	8.0772768	1.0429266	9.4344820	0.9586407	8.2604805
CTDaD	8.6642460	0.9446059	9.2356805	0.9792477	-
TDaD	8.5805922	0.8044233	8.6707749	0.9692952	-
DaD	9.0129861	0.9666121	9.1183652	-	-

Table 11. The statistics  $\ell$ , AIC, CAIC, BIC and HQIC for dataset III

Distribution	$\hat{\ell}$	AIC	CAIC	BIC	HQIC	Ranks
APCTDaD	435.1453	880.2905	881.1996	891.6738	884.8222	1 <sup>st</sup>
CTDaD	438.1489	884.2979	884.8949	893.4045	887.9233	2 <sup>nd</sup>
TDaD	444.912	897.824	898.421	906.9307	901.4494	3 <sup>rd</sup>
DaD	454.7097	915.4195	915.7724	922.2495	918.1385	4 <sup>th</sup>

A summary of dataset III in Table 9 and Fig. 11 has shown that it is a skewed data and therefore requires a skewed and flexible probability distribution for its proper analysis.

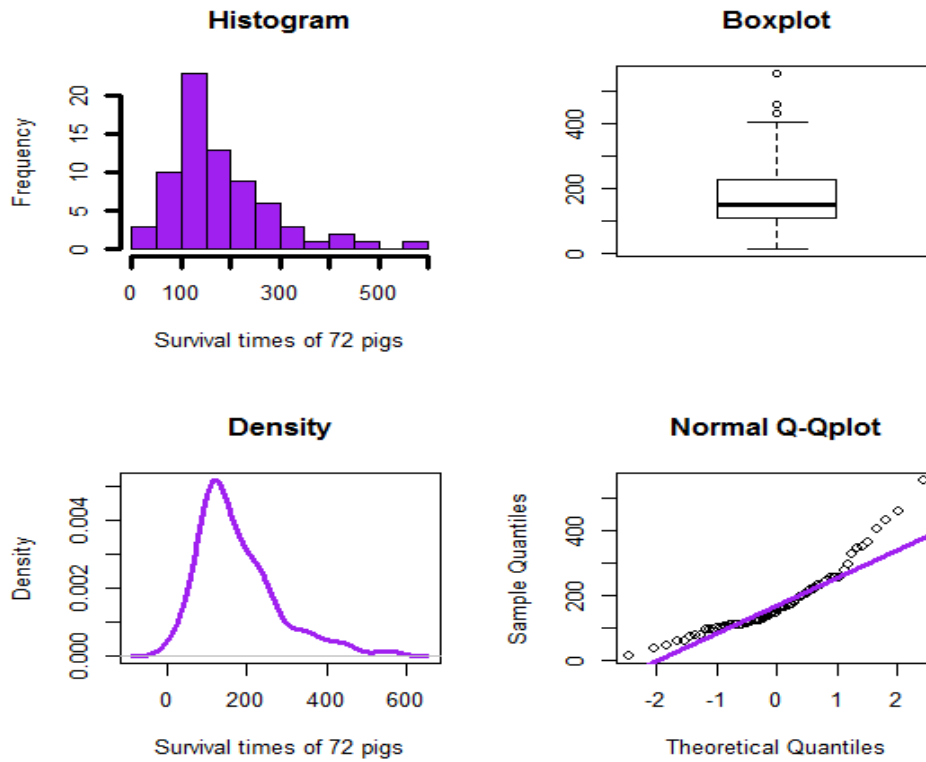


Fig. 11. A graphical summary of the dataset III

Table 12. The  $A^*$ ,  $W^*$ , K-S statistic and P-values for dataset III

Distribution	$A^*$	$W^*$	K-S	P-Value (K-S)	Ranks
APCTDaD	1.059438	0.1451422	0.16932	0.03223	1 <sup>st</sup>
CTDaD	1.442234	0.2061458	0.16971	0.03161	2 <sup>nd</sup>
TDaD	1.193157	0.1675743	0.19743	0.007299	3 <sup>rd</sup>
DaD	1.775791	0.259787	0.26276	9.62e-05	4 <sup>th</sup>

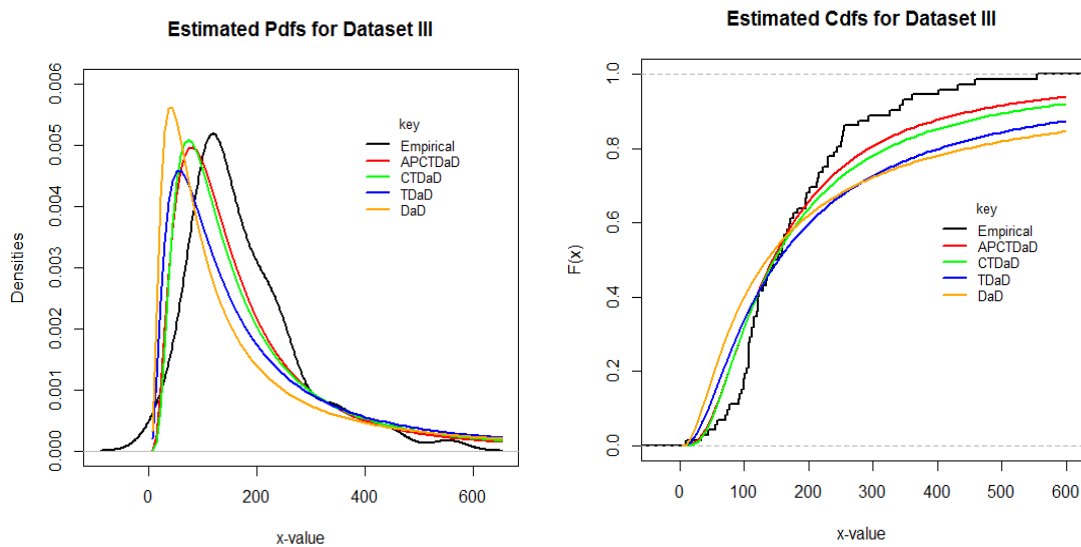
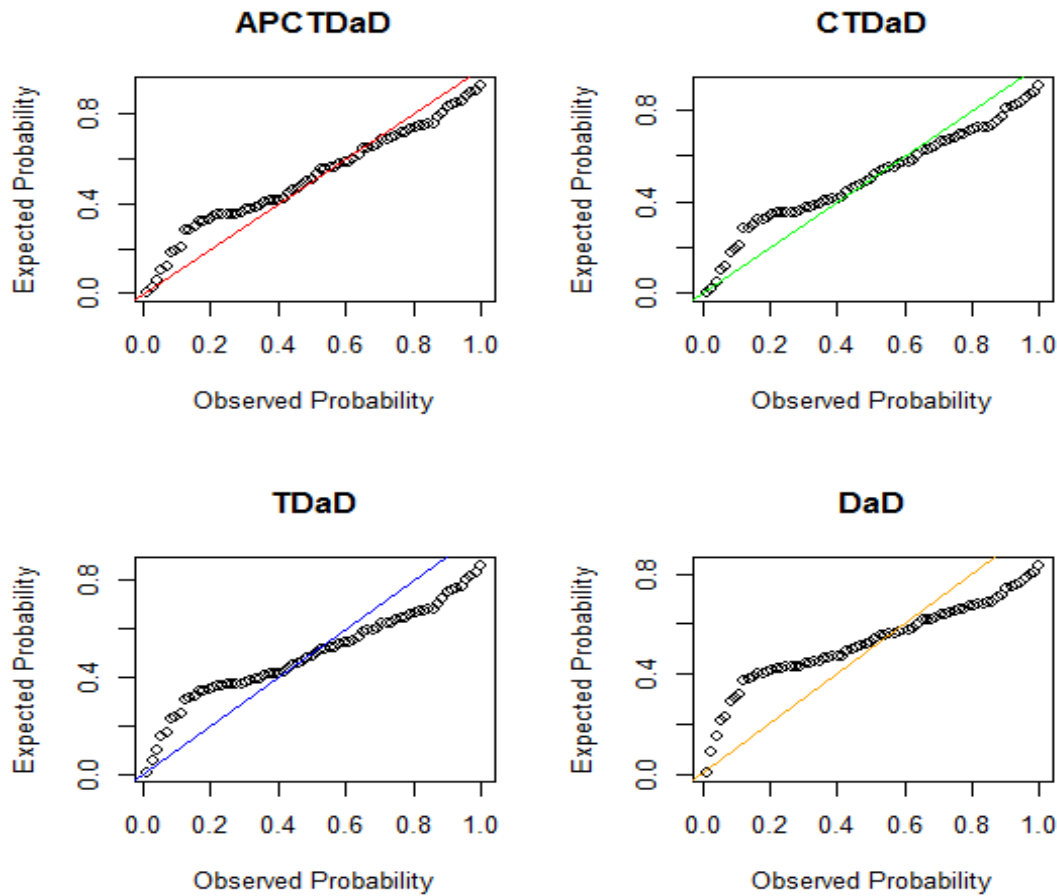


Fig. 12. Plots of the estimated densities and cdfs of the fitted distributions to dataset III



**Fig. 13. Probability plots for the fit of the APCTDaD, CTDaD, TDaD & DaD based on dataset III**

Tables 2, 6 and 10 present the MLEs of the fitted model parameters based on dataset I, II and III respectively, Table 3, 7 and 11 present the AIC, CAIC, BIC and HQIC values for dataset I, II and III respectively. Relatively, the values of  $A^*$ ,  $W^*$  and K-S for datasets I, II and III are given in Tables 4, 8 and 12 respectively.

The plots of the estimated density and cumulative distribution function of APCTDaD with those of the other fitted distributions are presented in Figs. 6, 9 and 12 for datasets I, II and III respectively. The probability plots of the fitted distributions are also given in Figs. 7, 10 and 13 for datasets I, II and III respectively. Considering the results from all the measures based on all the three datasets above, it is observed that the alpha power cubic transmuted Dagum distribution (APCTDaD) performs much better for all the real life datasets compared to the other three fitted distributions (cubic transmuted Dagum distribution (CTDaD), transmuted Dagum distribution (TDaD) and the conventional Dagum distribution (DaD)). These results are clearly confirmed by the estimated

density plots and also the probability plots of the fitted distributions as shown in the Figs. 7, 10 and 13 above. Considering all these results, it has concluded that the proposed distribution (alpha power cubic transmuted Dagum distribution (APCTDaD)) is a more flexible distribution than the other existing distributions fitted in this study. This study is in line with the fact that all extended distributions perform better than their classical counterpart and the research is also in agreement with the study of [13] and [16], which are also based on the alpha power transformation family.

#### 4. CONCLUSION

This research considered the alpha power transformation approach to define and study a Dagum distribution producing a new distribution called "alpha power cubic transmuted Dagum distribution (APCTDaD)". The research derived and studied the reliability functions of the proposed distribution which include survival and hazard functions with their graphical presentations and discussions on their

usefulness and applications. It is concluded that the proposed distribution is better compared to other existing distributions based on our application of the model to a real life datasets.

## COMPETING INTERESTS

Authors have declared that no competing interests exist.

## REFERENCES

- Dagum C. A new model of personal income distribution: Specification and estimation. *Economie Appliquee*. 1977;30: 413-437.
- Cordeiro GM, Ortega EMM, Ramires TG. A new generalized Weibull family of distributions: Mathematical properties and applications. *Journal of Statistical Distributions and Applications*. 2015;2: 13. DOI: 10.1186/s40488-015-0036-6
- Tahir MH, Cordeiro GM, Alzaatreh A, Mansoor M, Zubair M. The logistic-X family of distributions and its applications. *Communications in Statistics: Theory and Methods*. 2016;45(24):7326-7349.
- Tahir MH, Zubair M, Mansoor M, Cordeiro GM, Alizadeh M. A New Weibull-G family of distributions. *Hacettepe Journal of Mathematics and Statistics*. 2016;45(2): 629-647.
- Cakmakyapan S, Ozel G. The lindley family of distributions: Properties and Applications. *Hacettepe Journal of Mathematics and Statistics*. 2016;46:1-27.
- Alizadeh M, Cordeiro GM, Pinho LGB, Ghosh I. The Gompertz-G family of distributions. *Journal of Statistical Theory and Practice*. 2017;11(1):179–207. DOI: 10.1080/15598608.2016.1267668
- Gomes-Silva F, Percontini A, De Brito E, Ramos MW, Venancio R, Cordeiro GM. The Odd Lindley-G Family of Distributions. *Austrian Journal of Statistics*. 2017;46:65-87.
- Mahdavi A, Kundu D. A new method for generating distributions with an application to exponential distribution, *Communications in Statistics-Theory and Methods*. 2017;46(13):6543–6557.
- Cordeiro GM, Afify AZ, Ortega EMM, Suzuki AK, Mead ME. The odd Lomax generator of distributions: Properties, estimation and applications. *Journal of Computational and Applied Mathematics*. 2019;347:222–237.
- Tahir MH, Zubair M, Cordeiro GM, Alzaatreh A, Mansoor M. The Poisson-X family of distributions, *Journal of Statistical Computation and Simulation*, 2016;86(14): 2901-2921.
- Ieren TG, Balogun OS. Exponential-Lindley Distribution: Theory and Application to Bladder Cancer Data. *Journal of Applied Probability and Statistics*. 2021;16(2):129-146.
- Ieren TG, Abdullahi J. A Transmuted normal distribution: Properties and Applications. *Equity Journal of Science and Technology*. 2020;7(1):16-35.
- Nassar M, Alzaatreh A, Mead M, Abo-Kasem O. Alpha power weibull distribution: Properties and applications, *Communications in Statistics - Theory and Methods*. 2017;46(20):10236–10252.
- Ieren TG, Abdulkadir SS, Issa AA. Odd Lindley-Rayleigh Distribution: Its Properties and Applications to Simulated and Real Life Datasets. *Journal of Advances in Mathematics and Computer Science*. 2020;35(1):63-88.
- Umar SA, Bukar AB, Makama MS, Ieren TG. Some Results on the Transmuted Odd Lindley-Rayleigh Distribution. *Benin Journal of Statistics*. 2021;4:135– 153.
- Dey S, Alzaatreh A, Zhang C, Kumar D. A new extension of generalized exponential distribution with application to ozone data, *Ozone: Science & Engineering*. 2017; 39(4):273–285.
- Ieren TG, Abdullahi J. Properties and Applications of a Two-Parameter Inverse Exponential Distribution with a Decreasing Failure Rate. *Pakistan Journal of Statistics*. 2020;36(3):183-206.
- Chama AF. Cubic Transmuted Dagum Distribution: Properties and Applications. *Mathematics Letters*. 2021;7(1):7-18.
- Chen G, Balakrishnan N. A general purpose approximate goodness-of-fit test. *Journal of Quality Technology*. 1995;27: 154–161.
- Efron B. Logistic regression, survival analysis and the Kaplan-Meier curve. *Journal of the American Statistical Association*. 1988;83:414-425.
- Shanker R, Hagos F, Sujatha S. On modeling of lifetimes data using exponential and Lindley distributions. *Biometrics and Biostatistics International Journal*. 2015;2(5):140-147.

22. Oguntunde PE, Balogun OS, Okagbue HI, Bishop SA. The Weibull-Exponential Distribution: Its properties and application. Journal of Applied Sciences. 2015; 15(11):1305-1311.
23. Lee ET, Wang JW. Statistical Methods for Survival Data Analysis. 3rd Edn., John Wiley and Sons, New York. 2003;534. ISBN: 9780471458555.
24. Rady E, Hassanein A, WA, Elhaddad TA. The power Lomax distribution with an application to bladder cancer data. Springer Plus. 2016;5(1):1838.
25. Ieren TG, Chukwu AU. Bayesian Estimation of a Shape Parameter of the Weibull-Frechet Distribution. Asian J. of Prob. and Stat. 2018;2(1):1-19.
26. Abdullahi J, Abdullahi UK, Ieren TG, Kuhe DA, Umar AA. On the properties and applications of transmuted odd generalized exponential- exponential distribution, Asian J. of Prob. Stat. 2018;1(4):1-14.
27. Bjerkedal T. Acquisition of resistance in guinea pigs infected with different doses of virulent tubercle bacilli, American Journal of Hygiene. 1960;72:130-148.

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