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Decomposition with the Additive Model with Exponential Trend Curve and Detection of Seasonal Effect in Descriptive Time Series

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

This article examines decomposition with the additive model. The procedure of decomposition has involved the four basic components and requires a method that can adequately estimate and investigate the trend parameters, seasonal indices and residual component of the series. This article also consider test for seasonality in the additive model with exponential trend curve. The test is applied to the row and overall sample variances of the Buys-Ballot table to detect the presence of seasonal indices in time series.

Keywords: Buy-Ballot method; addive model; successful transformation; row variance; overall variance; exponential trend curve.

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1. INTRODUCTION

Classical decomposition method is equally known procedure of decomposing time series. Its applications is usually predicated on time series models. As the literature reveals, classical decomposition procedure has attracted so much research attention. The aims of the classical decomposition procedure have been mentioned in several studies. The advantages of the classical decomposition procedure are; it is used to investigate the presence of trend, seasonal, cyclical and error components in time series analysis. Time series analysis involve the separation of an observed series into components consisting trend (long term direction), seasonal (calendar related movements), cyclical (long term oscillations) and irregular (short term fluctuations).

The three time series models most commonly used are the

Additive Model:

$$X_t = T_t + S_t + C_t + I_t \quad (1)$$

Multiplicative Model:

$$X_t = T_t \times S_t \times C_t \times I_t \quad (2)$$

Mixed Model:

$$X_t = T_t \times S_t \times C_t + I_t \quad (3)$$

For short term period in which cyclical and trend components are jointly combined Chatfield [1] and the observed time series $\left(X_t, t = 1, 2, ..., n\right)$ can be decomposed into the trend-cycle component $\left(M_t\right)$, seasonal component $\left(S_t\right)$ and the irregular component $\left(e_t\right)$. Therefore, the decomposition models are

Additive Model:

$$X_t = M_t + S_t + e_t \quad (4)$$

Multiplicative Model:

$$X_t = M_t \times S_t \times e_t \quad (5)$$

and Mixed Model

$$X_t = M_t \times S_t + e_t. \quad (6)$$

Any of the decomposition models, additive or multiplicative or mixed model may be used to effect the decomposition of a time series. The procedure of decomposition has involved the four basic components which make up a time series analysis. In descriptive method of time series decomposition, the first step will normally be to estimate and then to eliminate trend-cycle (M_t) for each time period from the original data either by subtraction or division. The resulting time series after elimination the trend-cycle (M_t) is the de-trended series and expresses the effects of the season and irregular components. The de-trended series is expressed mathematically as:

$$X_t - \hat{M}_t \quad (7)$$

for the additive model or

$$X_t \,/\, \hat{M}_t \quad (8)$$

for the multiplicative model or

$$X_t \,/\, \hat{M}_t \quad (9)$$

for the mixed model

The seasonal effect is obtained by estimating the average of the de-trended series at each season. The de-trended, de-seasonalized series is obtained as

$$X_t - \hat{M}_t - \hat{S}_t \quad (10)$$

for the additive model, or

$$X_t \,/\left(\hat{M}_t\ \hat{S}_t\right) \quad (11)$$

for the multiplicative model

$$X_t \,/\left(\hat{M}_t\ \hat{S}_t\right) \quad (12)$$

for the mixed model

On when to use any of the three time series models, Chatfield [1] stated that, when the seasonal effect in direct proportion with the average, then the seasonal effect is be multiplicative model given in equation (2) may be employed. On the other hand, additive model given in equation (1) is applied, if the seasonal indices stays roughly the same size, regardless of the mean level. Nwogu, et al, [2] and Dozie, et al, [3] proposed a test for choice of model based on Chi-Square distribution. Although time series data does not satisfy all the assumptions of most common statistical test, the Chi-Square test appears to be the most efficient among. To choose the appropriate models among additive, multiplicative and mixed, many scholars have suggested different approaches. Puerto and Rivera [4] provided the use of coefficient of variation of seasonal differences CV(d) and seasonal quotient CV(c) for choice of model. According to them, additive model is appropriate, if CV(c) is greater than CV(d) and it is multiplicative if CV(c) is less than or equal to CV(d). However, they did not provide any statistical test to justify the use. Chatfield [1] suggested the use of time plot to choose between additive and multiplicative models. However, no theoretical basis was proposed for the decision rule. Iwueze, et al. [5] proposed the use of the relationship between the seasonal average ($\bar{X}_{.j}, j=1,2,...s$) and the seasonal standard deviations ($\hat{\sigma}_{.j}, j=1,2,...s$) to choose the appropriate model for decomposition.

Seasonality in time series data can be observed as a pattern that repeats every k elements. Some graphical methods are used to detect the presence of seasonal effect in time series are: (1) a run sequence plot (Chambers et al [6]). (2) a seasonal sub-series plot [7]; (3) multiple box plots [8]; Davey and Flores [9] proposed statistical tests for seasonality. Kendall and Ord [10] studied test of seasonality in time series analysis. Chatfield [1] presented the use of Buys-Ballot table for detecting the presence of trend and seasonal component in time series.

2. METHODOLOGY

Decomposition with the additive model when the trending curve is exponential and detection of seasonal effect are done using Buys-Ballot procedure often referred to in the literature. This method adopted in this study assumed that the series are arranged in a Buys-Ballot table with m rows and s columns. For details of this method see Wei [11], Iwueze et.al [5], Nwogu et.al [2], Dozie et.al [3], Dozie and Nwanya [12], Dozie [13], Dozie and Ijeomah [14], Dozie and Uwaezuoke [15], Dozie and Ihekuna [16] Dozie and Ibebuogu [17], Dozie and Ihekuna [18], Dozie and Ibebuogu [19]

2.1 Exponential Trend Cycle and Seasonal Effects

For a series that has exponential trend, the row, column and overall means obtained by Iwueze and Nwogu [20] when the observations are complete are given in Table 1.

Table 1. Summary of row, column and overall means of a series when trend cycle is exponential: ($M_t = be^{ct}$)

Measure	Exponential trend-cycle component: $M_t = be^{ct}, \quad t=1,2,...n=ms$
$\bar{X}_{i.}$	$\frac{b}{s}\left(\frac{e^{(1-s)}-e^{c}}{1-e^{c}}\right)e^{csi}$
$\bar{X}_{.j}$	$\frac{b}{m}\left(\frac{1-e^{cn}}{1-e^{cs}}\right)e^{cj}+s_j$
$\bar{X}_{..}$	$\frac{be^{c}}{n}\left(\frac{1-e^{cn}}{1-e^{c}}\right)$

Source: Iweze and Nwogu (2014)

Where

$$\hat{X}_{ij} = \hat{b}\ell^{\hat{c}[(i-1)s+j]} + \hat{S}_j + \hat{e}_{ij} \quad (13)$$

Using the row, column and overall means, they gave the following expressions for intercept, slop and seasonal indices

$$\hat{C} = \frac{C'}{S} \quad (14)$$

$$\hat{b} = b' \ell^{c\left(\frac{s-1}{2}\right)} \quad (15)$$

$$\hat{S}_{.j} = \bar{X}_{.j} - \bar{X}_{..} \quad (16)$$

where

$$\bar{X}_{.j} = \frac{b}{m}\left(\frac{1-e^{cn}}{1-e^{cs}}\right)e^{cj}+s_j \quad (17)$$

and

$$\bar{X}_{..} = \frac{be^{c}}{n}\left(\frac{1-e^{cn}}{1-e^{c}}\right) \tag{18}$$

2.2 Test for Seasonality in the Additive Model

2.2.1 Row and overall sample variances

The Buys-Ballot estimates for row and overall variances are listed in equations (19) and (20) respectively.

2.3 Exponential Trending Curve (be^{ct})

$$\sigma_{i.}^{2} = b^{2}e^{2c[(i-1)s+1]}\left[\left(\frac{1-e^{2cs}}{1-e^{2c}}\right)-\frac{1}{s}\left(\frac{1-e^{cs}}{1-e^{c}}\right)\right]+\sum_{j=1}^{s}S_{j}^{2}+2be^{c(i-1)s}\sum_{j=1}^{s}e^{cj}S_{j} \tag{19}$$

$$\sigma_{..}^{2} = \frac{b^{2}e^{2c}}{n-1}\left[\left(\frac{1-e^{2cn}}{1-e^{2c}}\right)-\frac{1}{n}\left(\frac{1-e^{cn}}{1-e^{c}}\right)^{2}\right]+\frac{m}{m-1}\sum_{j=1}^{s}S_{j}^{2}+\frac{2b}{n-1}\sum_{j=1}^{s}e^{cj}S_{j} \tag{20}$$

2.4 Choice of Appropriate Transformation

For time series data arranged in Buys-Ballot table Akpanta and Iwueze [21] provided the slope of the regression equation of log of group standard deviation on log of group mean as stated in equation (21) is what is needed for choice of appropriate transformation. Some of the values of slope β and their implied transformation are stated in Table 2.

$$\log_{e}\hat{\sigma}_{i} = a+\beta\log_{e}\hat{X}_{i.} \tag{21}$$

The method of Akpanta and Iwueze [21] is used in selecting the appropriate transformation, the natural logarithm of standard deviation will be used to regress against the natural logarithm of periodic means and the result of the β - value will determine the type of transformation.

3. EMPIRICAL EXAMPLE

This section presents empirical example to illustrate the application of the Buys-Ballot procedure discussed in chapter 2. One hundred and twenty (120) of reported cases of registered infant baptism at St Jude Church Amuzi, Mbaise in Imo State, Nigeria from January, 2012 to December, 2021. The natural logarithm of the periodic and standard deviation are given in Table 3. The data is transformed by using the natural logarithm of the one hundred and twenty (120) observations by the method of Akpanta and Iwueze [21]. The row and column totals, means and standard deviations of the Buys-Ballot table are given in Tables 5 and 6 respectively.

3.1 Estimates of Exponential Trend-Cycle and Seasonal Effect

Using equations (14), (15) and (16) we obtain b´ = 1.758 and c´ = 1.001

$$\hat{C} = \frac{1.001}{12} = 0.0834$$

$$\hat{b} = 1.758\times\ell^{0.0834\left(\frac{12-1}{2}\right)} = 1.758\times\ell^{0.4587} = 1.758\times1.5820 = 2.781$$

$$\hat{S}_{.j} = \bar{X}_{.j} - \bar{X}_{..}$$

Table 2. Bartlett's transformation for some values of β

S/No	1	2	3	4	5	6	7
β	**0**	$\frac{1}{2}$	**1**	$\frac{3}{2}$	**2**	**3**	**-1**
Transformation	**No transformation**	$\sqrt{X_t}$	$\log_e X_i$	$\frac{1}{\sqrt{X_t}}$	$\frac{1}{X_t}$	$\frac{1}{X_t^2}$	X_t^2

Table 3. Natural logarithm of periodic averages and standard deviations

$\bar{X}_{i.}$	$Log_e \bar{X}_{i.}$	$\hat{\sigma}_{i.}$	$Log_e \hat{\sigma}_{i.}$
9.00	2.20	4.69	1.55
6.82	1.92	2.48	0.91
6.67	1.90	2.84	1.04
6.42	1.86	3.58	1.28
7.67	2.04	3.70	1.31
7.08	1.96	4.12	1.46
9.83	2.29	4.63	1.53
8.92	2.19	3.60	1.28
7.33	1.99	5.84	1.76
9.50	2.25	3.92	1.37

Table 4. Estimates of seasonal effect

j	$\bar{X}_{.j}$	$\hat{S}_{.j}$	$Adj\ \hat{S}_j$
1	1.58	-0.27	-0.27
2	1.82	-0.03	-0.02
3	1.94	0.10	0.10
4	1.87	0.02	0.02
5	1.84	-0.01	-0.01
6	1.73	-0.12	-0.11
7	1.96	0.11	0.11
8	1.81	-0.03	-0.03
9	1.82	-0.02	-0.02
10	1.86	0.01	0.01
11	2.06	0.21	0.21
12	1.86	0.01	0.01
$\sum_{j=1}^{12} \hat{S}_{.j}$		-0.02	0.00

Table 5. Seasonal effect

Parameters	Seasonal effect
$\hat{b}$	1.76
$\hat{c}$	1.00
$\hat{S}_1$	-027
$\hat{S}_2$	-0.02

Parameters	Seasonal effect
$\hat{S}_3$	0.10
$\hat{S}_4$	0.02
$\hat{S}_5$	0.01
$\hat{S}_6$	-0.11
$\hat{S}_7$	0.11
$\hat{S}_8$	-0.03
$\hat{S}_9$	-0.02
$\hat{S}_{10}$	0.01
$\hat{S}_{11}$	0.21
$\hat{S}_{12}$	0.01

Table 6. Row totals, means and variances

Periods				
i	r_i	T_i	$\bar{X}_i$	$\hat{\sigma}_i^2$
1	10	21.15	1.92	0.97
2	10	19.72	1.79	1.79
3	10	21.20	1.77	0.25
4	10	20.55	1.71	0.35
5	10	22.05	1.84	0.28
6	10	21.23	1.77	0.34
7	10	23.82	1.97	0.30
8	10	23.26	1.94	0.25
9	10	21.19	1.77	0.39
10	10	23.65	1.97	0.28
Overall Total	100	217.82	1.85	0.30

$$n = \sum_{j=1}^{r} c_j = \sum_{i=1}^{c} r_i = total\ number\ of\ observation$$

Where,

r_i = Number of observation in the r[th] row

c_j = Number of observation in the j[th] column.

Table 7. Column totals, means and variances

Seasons				
j	c_j	$T._j$	$\bar{X}_{.j}$	$\hat{\sigma}_{.j}^2$
1	12	14.18	1.58	0.14
2	12	18.21	1.82	0.28
3	12	19.43	1.94	0.26
4	12	18.65	1.87	0.32
5	12	18.35	1.84	0.32

Seasons				
j	c_j	$T_{\cdot j}$	$\bar{X}_{.j}$	$\hat{\sigma}^2_{.j}$
6	12	17.30	1.73	0.24
7	12	17.60	1.96	0.35
8	12	18.14	1.81	0.34
9	12	18.23	1.82	0.27
10	12	18.58	1.86	0.33
11	12	20.59	2.06	0.26
12	12	18.58	1.86	0.39
Overall Total	144	217.82	1.85	0.30

3.2 Application of Test for Seasonality in the Buys-Ballot table

Matched pairs of data are applied to the row and overall variances of the Buys-Ballot For the matched pairs of data, $(X_i, Y_i),\ \ i = 1, 2, ...n$, define $d_i = X_i - Y_i$. For identification of the presence and absence of seasonal indices in time series data. Let X_i represents row and overall variances in the presence of seasonal indices and denote Y_i represents row and overall variances in the absence of seasonal indices (Nwogu *et al.* [22]).

3.3 For Exponential Trend, in the Presence of Seasonal Effect, the Row Variance is Obtained as

$$X_i(E) = \sigma^2_{i.}(E) = b^2 e^{2c[(i-1)s+1]}\left[\left(\frac{1-e^{2cs}}{1-e^{2c}}\right) - \frac{1}{s}\left(\frac{1-e^{cs}}{1-e^{c}}\right)\right] + \sum_{j=1}^{s} S_j^2 + 2be^{c(i-1)s}\sum_{j=1}^{s} e^{cj} S_j \quad (22)$$

When there is no seasonal indices, $S_j = 0 \forall j = 1, 2, ..., s,$ therefore

$$Y_i(E) = b^2 e^{2c[(i-1)s+1]}\left[\left(\frac{1-e^{2cs}}{1-e^{2c}}\right) - \frac{1}{s}\left(\frac{1-e^{cs}}{1-e^{c}}\right)\right] \quad (23)$$

$$d_i(E) = X_i(E) - Y_i(E) = \sum_{j=1}^{s} S_j^2 + 2be^{c(i-1)s}\sum_{j=1}^{s} e^{cj} S_j \quad (24)$$

Which is zero under null hypothesis ($H_o : S_j = 0$)

3.4 The Overall Variance is Obtained as

$$X_i(E) = \sigma^2_{..}(E) = \frac{b^2 e^{2c}}{n-1}\left[\left(\frac{1-e^{2cn}}{1-e^{2c}}\right) - \frac{1}{n}\left(\frac{1-e^{cn}}{1-e^{c}}\right)^2\right] + \frac{m}{m-1}\sum_{j=1}^{s} S_j^2 + \frac{2b}{n-1}\sum_{j=1}^{s} e^{cj} S_j$$

(25)

When there is no seasonal indices, $S_j = 0 \forall j = 1, 2, ..., s,$ therefore

$$Y_i(E) = \frac{b^2 e^{2c}}{n-1}\left[\left(\frac{1-e^{2cn}}{1-e^{2c}}\right) - \frac{1}{n}\left(\frac{1-e^{cn}}{1-e^{c}}\right)^2\right] \quad (26)$$

$$d_i = X_i(E) - Y_i(E) = \frac{m}{m-1}\sum_{j=1}^{s} S_j^2 + \frac{2b}{n-1}\sum_{j=1}^{s} e^{cj} S_j \qquad (27)$$

Which is zero under null hypothesis ($H_o : S_j = 0$)

Table 8. Estimates in the presence of seasonal effect for row variance

Exponential Trending Curve (be^{ct})	$\sum_{j=1}^{s} S_j^2 + 2be^{c(i-1)s}\sum_{j=1}^{s} e^{cj} S_j$

Table 9. Estimates in the absence of seasonal effect for row variance

Exponential Trending Curve (be^{ct})	$b^2 e^{2c[(i-1)s+1]}\left[\left(\frac{1-e^{2cs}}{1-e^{2c}}\right) - \frac{1}{s}\left(\frac{1-e^{cs}}{1-e^{c}}\right)\right]$

Table 10. Estimates in the presence of seasonal effect for overall variance

Exponential Trending Curve (be^{ct})	$\frac{m}{m-1}\sum_{j=1}^{s} S_j^2 + \frac{2b}{n-1}\sum_{j=1}^{s} e^{cj} S_j$

Table 11. Estimates in absence of seasonal effect for overall variance

Exponential Trending Curve (be^{ct})	$\frac{b^2 e^{2c}}{n-1}\left[\left(\frac{1-e^{2cn}}{1-e^{2c}}\right) - \frac{1}{n}\left(\frac{1-e^{cn}}{1-e^{c}}\right)^2\right]$

Additive seasonality in time series is employed using matched pairs of data in the Buys-Ballot table for exponential trend curve shown in equations (19) and (20). The test has been developed using the row and overall variances of the Buys-Ballot table. The estimates for the data in the presence of seasonal indices for row and overall variances are listed in Tables 8 and 10 respectively. While the estimates for series without seasonal indices for periodic and overall variances are given in Tables 9 and 11. The Buys-Ballot estimates obtained and listed in equations (24) and (27) are products of seasonal effects only when the trend parameters are removed, while that of equations (23) and (26) are functions of trend parameters.

4. SUMMARY, CONCLUSION AND RECOMMENDATIONS

This study has examined decomposition with the additive model and test of seasonality in time series. The method adopted in this study is Buys-Ballot procedure developed for choice of model among other uses based on row, column and overall means and variances of the Buys-Ballot table. This study is limited to a series in when trend-cycle component is exponential and admits additive model. This study also consider a test for seasonality in the additive model with exponential trend curve. The test is applied to the row and overall sample variances of the Buys-Ballot table to detect the presence of seasonal indices. Results indicate that, the Buys-Ballot estimates obtained and listed in equations (24) and (27) are functions of seasonal indices only, while that of equations (23) and (26) are functions of trend parameters. This study has provided decomposition with the additive model with exponential trend curve. Other trending curves yet to be considered, are therefore recommended for further investigation.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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Appendix A. Buys ballot table of the actual data on the number of infant

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{y}_{i.}$	$\sigma_{i.}$
2012	4.0	10.0	9.0	15.0	18.0	7.0	10.0	3.0	13.0	7.0	8.0	5.0	99.0	9.00	4.69
2013	4.0	7.0	6.0	6.0	5.0	6.0	12.0	4.0	5.0	7.0	11.0	6.0	75.0	6.82	2.48
2014	4.0	7.0	5.0	10.0	6.0	2.0	7.0	5.0	8.0	6.0	7.0	13.0	80.0	6.67	2.84
2015	3.0	8.0	12.0	7.0	6.0	8.0	10.0	11.0	2.0	2.0	6.0	2.0	77.0	6.42	3.58
2016	4.0	3.0	9.0	5.0	5.0	6.0	5.0	7.0	9.0	12.0	13.0	14.0	92.0	7.67	3.70
2017	3.0	2.0	14.0	5.0	3.0	9.0	5.0	12.0	10.0	4.0	12.0	6.0	85.0	7.08	4.12
2018	6.0	9.0	16.0	13.0	10.0	9.0	19.0	3.0	7.0	7.0	6.0	13.0	118.0	9.83	4.63
2019	7.0	8.0	6.0	4.0	9.0	6.0	13.0	10.0	5.0	11.0	12.0	16.0	107.0	8.92	3.60
2020	4.0	9.0	5.0	3.0	4.0	6.0	3.0	5.0	8.0	17.0	21.0	3.0	88.0	7.33	5.84
2021	5.0	11.0	8.0	14.0	12.0	3.0	12.0	16.0	7.0	8.0	12.0	6.0	114.0	9.50	3.92
total	40.0	74.0	90.0	82.0	78.0	62.0	86.0	76.0	74.0	81.0	108.0	84.0	935		
$\bar{y}_{.j}$	4.44	7.40	9.00	8.20	7.80	6.20	9.56	7.60	7.40	8.10	10.80	8.40		7.92	
$\sigma_{.j}$	1.33	2.88	3.86	4.44	4.57	2.30	5.05	4.43	3.03	4.28	4.49	5.06			4.07

Appendix B. Buys-ballot of transformed data

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{y}_{i.}$	$\sigma_{i.}$
2012	1.537	2.042	1.976	2.315	2.450	1.828	5.097	1.406	2.215	1.828	1.905	1.647	26.245	2.187	0.967
2013	7.954	1.828	1.743	1.743	1.647	1.743	2.160	1.537	1.647	1.828	2.103	1.743	27.675	2.306	1.788
2014	1.537	1.828	1.647	2.042	1.743	1.240	1.828	1.647	1.905	1.743	1.828	2.215	21.202	1.767	0.246
2015	1.406	1.905	2.160	1.828	1.743	1.905	2.042	2.103	1.240	1.240	1.743	1.240	20.554	1.713	0.346
2016	1.537	1.406	1.976	1.647	1.647	1.743	1.647	1.828	1.976	2.160	2.215	2.266	22.048	1.837	0.280
2017	1.406	1.240	2.266	1.647	1.406	1.976	1.647	2.160	2.042	1.537	2.160	1.743	21.230	1.769	0.344
2018	1.743	1.976	2.362	2.215	2.042	1.976	2.491	1.406	1.828	1.828	1.743	2.215	23.824	1.985	0.303
2019	1.828	1.905	1.743	1.537	1.976	1.743	2.215	2.042	1.647	2.103	2.160	2.362	23.261	1.938	0.249
2020	1.537	1.976	1.647	1.406	1.537	1.743	1.406	1.647	1.905	2.407	2.570	1.406	21.186	1.765	0.387
2021	1.647	2.103	1.905	2.266	2.160	1.406	2.160	2.362	1.828	1.905	2.160	1.743	23.647	1.971	0.281
total	22.131	18.209	19.426	18.645	18.350	17.302	22.693	18.137	18.233	18.579	20.588	18.578	230.870		
$\bar{y}_{.j}$	2.213	1.821	1.943	1.865	1.835	1.730	2.269	1.814	1.823	1.858	2.059	1.858		1.924	
$\sigma_{.j}$	2.022	0.279	0.255	0.324	0.318	0.238	1.046	0.337	0.267	0.326	0.257	0.387			0.698

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