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A Quarter-Step Hybrid Block Method for First-Order Ordinary Differential Equations

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Abstract

In this paper, we derive a new quarter-step hybrid block method for the solution of first-order Ordinary Differential Equations (ODEs). We employ the approach of interpolating the power series and collocating the differential system within a quarter-step interval of integration. The evaluation is carried out at off grid points within the step of the method to produce various discrete schemes to form our block method. The basic properties of the new hybrid block method were further investigated. The new method was also tested on some problems and the results obtained were found to compete favorably with those of the existing ones.

Keywords: Approximate solution, collocation, hybrid, interpolation, quarter-step.

2010 AMS Subject Classification: 65L05, 65L06, 65D30.

1 Introduction

In this paper, we consider the numerical solution of first-order ordinary differential equations of the form,

$$y' = f(x, y),\ y(x_0) = y_0,\ a \le x \le b \tag{1}$$

where $f : \Re \times \Re^m \to \Re^m, y, y_0 \in \Re^m$, f is assumed to satisfy Lipchitz condition.

Most of the problems in Sciences, Medicine, Agriculture, e.t.c. are modeled in the form of (1), the few that are modeled in higher order are first reduced to systems of first order before appropriate

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method of solution is applied, [1]. Scholars have proposed different numerical methods for the solution of (1), these methods can be in the form of single step or multistep methods. Multistep method can be in the form of k-step method or hybrid method. Hybrid method has been reported to have circumvented Dahlquist barrier condition through the introduction of off step points, though this method is difficult to develop but it gives better approximation than the k-step method especially when the method is of low step-length, [2]. Hybrid method is equally reported to give better stability condition especially when the problem is stiff or oscillatory, [3], [4] and [5].

In this paper, we develop a new method called the quarter-step method which gives results at a non-overlapping interval. The paper is organized as follows; introduction has been given in section one, section two discusses the derivation of the new method, in section three we analyze the basic properties of the method derived. Section four considers the numerical experiments and the discussion of results. Finally, section five gives conclusion and necessary recommendations.

Theorem 1 [1]: *Let $f(x,y)$ be defined and continuous for all points (x,y) in the region D defined by $a \le x \le b,\ -\infty < y < \infty$, a and b finite, and let there exists a constant L such that, for every x, y, y^* such that (x,y) and (x,y^*) are both in D;*

$$\left| f(x,y) - f(x,y^*) \right| \le L \left| y - y* \right|$$

Then, if y_0 is any given number, there exists a unique solution $y(x)$ of the initial value problem (1), where $y(x)$ is continuous and differentiable for all (x,y) in D.

2 Methodology

2.1 Derivation Technique of the Quarter-step Method

Consider the power series approximate solution of the form;

$$y(x) = \sum_{j=0}^{r+s-1} a_j x^j \tag{2}$$

where r and s are the number of interpolation and collocation points respectively. The first derivative of (2) is given by,

$$y'(x) = \sum_{j=1}^{r+s-1} j a_j x^{j-1} \tag{3}$$

where $a_j \in \Re$ for $j = 0(1)7$ and $y(x)$ is continuously differentiable. Let the solution of (1) be sought on the integration interval $[a,b]$ with a constant step-size h, defined by, $h = x_{n+1} - x_n$, $n = 0,1,...,N$

Substituting equation (3) in (1) gives,

$$f(x,y)=\sum_{j=0}^{r+s-1} ja_j x^{j-1} \tag{4}$$

We interpolate equation (2) at point $x_{n+s}, s=0$ and collocate equation (4) at points $x_{n+r}, r=\left[0,\frac{1}{24},\frac{1}{8},\frac{1}{6},\frac{5}{24},\frac{1}{4}\right]$. This leads to the following system of equations,

$$XA=U \tag{5}$$

where

$$A=[a_0\ \ a_1\ \ a_2\ \ a_3\ \ a_4\ \ a_5\ \ a_6\ \ a_7]^T$$

$$U=\left[y_n\ \ f_n\ \ f_{n+\frac{1}{24}}\ \ f_{n+\frac{1}{12}}\ \ f_{n+\frac{1}{8}}\ \ f_{n+\frac{1}{6}}\ \ f_{n+\frac{5}{24}}\ \ f_{n+\frac{1}{4}}\right]^T$$

and

$$X=\begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 \\ 0 & 1 & 2x_{n+\frac{1}{24}} & 3x^2_{n+\frac{1}{24}} & 4x^3_{n+\frac{1}{24}} & 5x^4_{n+\frac{1}{24}} & 6x^5_{n+\frac{1}{24}} & 7x^6_{n+\frac{1}{24}} \\ 0 & 1 & 2x_{n+\frac{1}{12}} & 3x^2_{n+\frac{1}{12}} & 4x^3_{n+\frac{1}{12}} & 5x^4_{n+\frac{1}{12}} & 6x^5_{n+\frac{1}{12}} & 7x^6_{n+\frac{1}{12}} \\ 0 & 1 & 2x_{n+\frac{1}{8}} & 3x^2_{n+\frac{1}{8}} & 4x^3_{n+\frac{1}{8}} & 5x^4_{n+\frac{1}{8}} & 6x^5_{n+\frac{1}{8}} & 7x^6_{n+\frac{1}{8}} \\ 0 & 1 & 2x_{n+\frac{1}{6}} & 3x^2_{n+\frac{1}{6}} & 4x^3_{n+\frac{1}{6}} & 5x^4_{n+\frac{1}{6}} & 6x^5_{n+\frac{1}{6}} & 7x^6_{n+\frac{1}{6}} \\ 0 & 1 & 2x_{n+\frac{5}{24}} & 3x^2_{n+\frac{5}{24}} & 4x^3_{n+\frac{5}{24}} & 5x^4_{n+\frac{5}{24}} & 6x^5_{n+\frac{5}{24}} & 7x^6_{n+\frac{5}{24}} \\ 0 & 1 & 2x_{n+\frac{1}{4}} & 3x^2_{n+\frac{1}{4}} & 4x^3_{n+\frac{1}{4}} & 5x^4_{n+\frac{1}{4}} & 6x^5_{n+\frac{1}{4}} & 7x^6_{n+\frac{1}{4}} \end{bmatrix}$$

Solving (5), for the $a_j\text{'}s, j=0(1)7$ and substituting back into (2) gives a linear multistep hybrid method of the form:

$$y(t)=\alpha_0(t)y_n+h\begin{bmatrix} \beta_0(t)f_n+\beta_{\frac{1}{24}}(t)f_{n+\frac{1}{24}}+\beta_{\frac{1}{12}}(t)f_{n+\frac{1}{12}} \\ +\beta_{\frac{1}{8}}(t)f_{n+\frac{1}{8}}+\beta_{\frac{1}{6}}(t)f_{n+\frac{1}{6}}+\beta_{\frac{5}{24}}(t)f_{n+\frac{5}{24}}+\beta_{\frac{1}{4}}(t)f_{n+\frac{1}{4}} \end{bmatrix} \tag{6}$$

where

$$\left.\begin{aligned}
&\alpha_0 = 1\\
&\beta_0 = \frac{1}{105}(3981312t^7 - 4064256t^6 + 1693440t^5 - 370440t^4 + 45472t^3 - 3087t^2 + 105t)\\
&\beta_{\frac{1}{24}} = -\frac{1}{35}(7962624t^7 - 7741440t^6 + 2999808t^5 - 584640t^4 + 58464t^3 - 2520t^2)\\
&\beta_{\frac{1}{12}} = \frac{1}{35}(19906560t^7 - 18385920t^6 + 6628608t^5 - 1161720t^4 + 98280t^3 - 3150t^2)\\
&\beta_{\frac{1}{8}} = -\frac{1}{105}(79626240t^7 - 69672960t^6 + 23417856t^5 - 3749760t^4 + 284480t^3 - 8400t^2)\\
&\beta_{\frac{1}{6}} = \frac{1}{35}(19906560t^7 - 16450560t^6 + 5177088t^5 - 773640t^4 + 55440t^3 - 1575t^2)\\
&\beta_{\frac{5}{24}} = -\frac{1}{35}(7962624t^7 - 6193152t^6 + 1838592t^5 - 262080t^4 + 18144t^3 - 504t^2)\\
&\beta_{\frac{1}{4}} = \frac{1}{105}(3981312t^7 - 2903040t^6 + 822528t^5 - 113400t^4 + 7672t^3 - 210t^2)
\end{aligned}\right\} \tag{7}$$

where $t = (x - x_n)/h$. Evaluating (6) at $t = \left[\frac{1}{24}, \frac{1}{12}, \frac{1}{8}, \frac{1}{6}, \frac{5}{24}, \frac{1}{4}\right]$ gives a discrete block scheme of the form:

$$A^{(0)}\mathbf{Y}_m = \mathbf{E}\mathbf{y}_n + hd\mathbf{f}(\mathbf{y}_n) + hb\mathbf{F}(\mathbf{Y}_m) \tag{8}$$

where

$$\mathbf{Y}_m = \begin{bmatrix} y_{n+\frac{1}{24}} & y_{n+\frac{1}{12}} & y_{n+\frac{1}{8}} & y_{n+\frac{1}{6}} & y_{n+\frac{5}{24}} & y_{n+\frac{1}{4}} \end{bmatrix}^T, \quad \mathbf{y}_n = \begin{bmatrix} y_{n-\frac{5}{24}} & y_{n-\frac{1}{6}} & y_{n-\frac{1}{8}} & y_{n-\frac{1}{12}} & y_{n-\frac{1}{24}} & y_n \end{bmatrix}^T$$

$$\mathbf{F}(\mathbf{Y}_m) = \begin{bmatrix} f_{n+\frac{1}{24}} & f_{n+\frac{1}{12}} & f_{n+\frac{1}{8}} & f_{n+\frac{1}{6}} & f_{n+\frac{5}{24}} & f_{n+\frac{1}{4}} \end{bmatrix}^T, \quad \mathbf{f}(\mathbf{y}_n) = \begin{bmatrix} f_{n-\frac{5}{24}} & f_{n-\frac{1}{6}} & f_{n-\frac{1}{8}} & f_{n-\frac{1}{12}} & f_{n-\frac{1}{24}} & f_n \end{bmatrix}^T$$

$$A^{(0)} = \begin{bmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} 0&0&0&0&0&1\\ 0&0&0&0&0&1\\ 0&0&0&0&0&1\\ 0&0&0&0&0&1\\ 0&0&0&0&0&1\\ 0&0&0&0&0&1 \end{bmatrix}, \quad d = \begin{bmatrix} 0&0&0&0&0&\frac{19087}{1451520}\\ 0&0&0&0&0&\frac{1139}{90720}\\ 0&0&0&0&0&\frac{137}{10752}\\ 0&0&0&0&0&\frac{143}{11340}\\ 0&0&0&0&0&\frac{3715}{290304}\\ 0&0&0&0&0&\frac{41}{3360} \end{bmatrix}$$

$$b=\begin{bmatrix} \frac{2713}{60480} & \frac{-15487}{483840} & \frac{293}{11340} & \frac{-6737}{483840} & \frac{263}{60480} & \frac{-863}{1451520} \\ \frac{47}{756} & \frac{11}{30240} & \frac{83}{5670} & \frac{-269}{30240} & \frac{11}{3780} & \frac{-37}{90720} \\ \frac{27}{448} & \frac{387}{17920} & \frac{17}{420} & \frac{-243}{17920} & \frac{9}{2240} & \frac{-29}{53760} \\ \frac{58}{945} & \frac{16}{945} & \frac{9}{1120} & \frac{29}{3780} & \frac{2}{945} & \frac{-1}{2835} \\ \frac{725}{12096} & \frac{2125}{96768} & \frac{125}{2268} & \frac{3875}{96768} & \frac{235}{12096} & \frac{-275}{290304} \\ \frac{9}{140} & \frac{9}{1120} & \frac{17}{210} & \frac{9}{1120} & \frac{9}{140} & \frac{41}{3360} \end{bmatrix}$$

It is important to note that the quarter-step method has 6 function evaluations per step.

3 Analysis of Basic Properties of the Quarter-step Method

3.1 Order of the Quarter-step Method

Let the linear operator $L\{y(x);h\}$ associated with the block (8) be defined as,

$$L\{y(x);h\}=A^{(0)}Y_m-Ey_n-h^{\mu}df(y_n)-h^{\mu}bF(Y_m) \tag{9}$$

where μ is the order of the differential equation. Expanding (9) using Taylor series and comparing the coefficients of h gives,

$$L\{y(x);h\}=c_0y(x)+c_1hy'(x)+c_2h^2y''(x)+...+c_ph^py^p(x)+c_{p+1}h^{p+1}y^{p+1}(x)+... \tag{10}$$

Definition 3: The linear operator L and the associated linear multistep method (6) are said to be of order p if $c_0=c_1=c_2=...=c_p=0$ and $c_{p+1}\neq 0$, see [6]. c_{p+1} is called the error constant and the local truncation error is given by,

$$t_{n+k}=c_{p+1}h^{(p+1)}y^{(p+1)}(x_n)+O(h^{p+2}) \tag{11}$$

For our quarter-step method,

$$L\{y(x):h\}=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n+\frac{1}{24}}\\y_{n+\frac{1}{12}}\\y_{n+\frac{1}{8}}\\y_{n+\frac{1}{6}}\\y_{n+\frac{5}{24}}\\y_{n+\frac{1}{4}}\end{bmatrix}-\begin{bmatrix}0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n-\frac{5}{24}}\\y_{n-\frac{1}{6}}\\y_{n-\frac{1}{8}}\\y_{n-\frac{1}{12}}\\y_{n-\frac{1}{24}}\\y_n\end{bmatrix}-h\begin{bmatrix} \frac{19087}{1451520} & \frac{2713}{60480} & \frac{-15487}{483840} & \frac{293}{11340} & \frac{-6737}{483840} & \frac{263}{60480} & \frac{-863}{1451520} \\ \frac{1139}{90720} & \frac{47}{756} & \frac{11}{30240} & \frac{83}{5670} & \frac{-269}{30240} & \frac{11}{3780} & \frac{-37}{90720} \\ \frac{137}{10752} & \frac{27}{448} & \frac{387}{17920} & \frac{17}{420} & \frac{-243}{17920} & \frac{9}{2240} & \frac{-29}{53760} \\ \frac{143}{11340} & \frac{58}{945} & \frac{16}{945} & \frac{9}{1120} & \frac{29}{3780} & \frac{2}{945} & \frac{-1}{2835} \\ \frac{3715}{290304} & \frac{725}{12096} & \frac{2125}{96768} & \frac{125}{2268} & \frac{3875}{96768} & \frac{235}{12096} & \frac{-275}{290304} \\ \frac{41}{3360} & \frac{9}{140} & \frac{9}{1120} & \frac{17}{210} & \frac{9}{1120} & \frac{9}{140} & \frac{41}{3360} \end{bmatrix}\begin{bmatrix}f_n\\f_{n+\frac{1}{24}}\\f_{n+\frac{1}{12}}\\f_{n+\frac{1}{8}}\\f_{n+\frac{1}{6}}\\f_{n+\frac{5}{24}}\\f_{n+\frac{1}{4}}\end{bmatrix} \tag{12}$$

Expanding (12) in Taylor series gives,

$$\begin{bmatrix} \sum_{j=0}^{\infty}\frac{\left(\frac{1}{24}h\right)^j}{j!}y_n^j - y_n - \frac{19087h}{1451520}y_n' - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}y_n^{j+1}\left\{\frac{2713}{60480}\left(\frac{1}{24}\right)^j - \frac{15487}{483840}\left(\frac{1}{12}\right)^j + \frac{293}{11340}\left(\frac{1}{8}\right)^j - \frac{6737}{483840}\left(\frac{1}{6}\right)^j + \frac{263}{60480}\left(\frac{5}{24}\right)^j - \frac{863}{1451520}\left(\frac{1}{4}\right)^j\right\} \\ \sum_{j=0}^{\infty}\frac{\left(\frac{1}{12}h\right)^j}{j!}y_n^j - y_n - \frac{1139h}{90720}y_n' - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}y_n^{j+1}\left\{\frac{47}{756}\left(\frac{1}{24}\right)^j + \frac{11}{30240}\left(\frac{1}{12}\right)^j + \frac{83}{5670}\left(\frac{1}{8}\right)^j - \frac{269}{30240}\left(\frac{1}{6}\right)^j + \frac{11}{3780}\left(\frac{5}{24}\right)^j - \frac{37}{90720}\left(\frac{1}{4}\right)^j\right\} \\ \sum_{j=0}^{\infty}\frac{\left(\frac{1}{8}h\right)^j}{j!}y_n^j - y_n - \frac{137h}{10752}y_n' - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}y_n^{j+1}\left\{\frac{27}{448}\left(\frac{1}{24}\right)^j + \frac{387}{17920}\left(\frac{1}{12}\right)^j + \frac{17}{420}\left(\frac{1}{8}\right)^j - \frac{243}{17920}\left(\frac{1}{6}\right)^j + \frac{9}{2240}\left(\frac{5}{24}\right)^j - \frac{29}{53760}\left(\frac{1}{4}\right)^j\right\} \\ \sum_{j=0}^{\infty}\frac{\left(\frac{1}{6}h\right)^j}{j!}y_n^j - y_n - \frac{143h}{11340}y_n' - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}y_n^{j+1}\left\{\frac{58}{945}\left(\frac{1}{24}\right)^j + \frac{16}{945}\left(\frac{1}{12}\right)^j + \frac{9}{1120}\left(\frac{1}{8}\right)^j + \frac{29}{3780}\left(\frac{1}{6}\right)^j + \frac{2}{945}\left(\frac{5}{24}\right)^j - \frac{1}{2835}\left(\frac{1}{4}\right)^j\right\} \\ \sum_{j=0}^{\infty}\frac{\left(\frac{5}{24}h\right)^j}{j!}y_n^j - y_n - \frac{3715h}{290304}y_n' - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}y_n^{j+1}\left\{\frac{725}{12096}\left(\frac{1}{24}\right)^j + \frac{2125}{96768}\left(\frac{1}{12}\right)^j + \frac{125}{2268}\left(\frac{1}{8}\right)^j + \frac{3875}{96768}\left(\frac{1}{6}\right)^j + \frac{235}{120304}\left(\frac{5}{24}\right)^j - \frac{275}{290304}\left(\frac{1}{4}\right)^j\right\} \\ \sum_{j=0}^{\infty}\frac{\left(\frac{1}{4}h\right)^j}{j!}y_n^j - y_n - \frac{41h}{3360}y_n' - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}y_n^{j+1}\left\{\frac{9}{140}\left(\frac{1}{24}\right)^j + \frac{9}{1120}\left(\frac{1}{12}\right)^j + \frac{17}{210}\left(\frac{1}{8}\right)^j + \frac{9}{1120}\left(\frac{1}{6}\right)^j + \frac{9}{140}\left(\frac{5}{24}\right)^j + \frac{41}{3360}\left(\frac{1}{4}\right)^j\right\} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (13)$$

Equating the coefficients of the Taylor series expansion to zero yields,

$$\bar{c}_0 = \bar{c}_1 = \bar{c}_2 = \bar{c}_3 = \bar{c}_4 = \bar{c}_5 = \bar{c}_6 = 0$$
$$\bar{c}_7 = \left[1.48(-011)\ \ 9.91(-012)\ \ 1.37(-011)\ \ 7.23(-012)\ \ 6.58(-011)\ \ 2.17(-010)\right]^T$$

Therefore, the quarter-step method is of uniform order 6.

3.2 Zero Stability of the Quarter-step Method

Definition 4 : *The block integrator (8) is said to be zero-stable, if the roots* $z_s, s = 1, 2, ..., k$ *of the first characteristic polynomial* $\rho(z)$ *defined by* $\rho(z) = \det(z\mathbf{A}^{(0)} - \mathbf{E})$ *satisfies* $|z_s| \le 1$ *and every root satisfying* $|z_s| \le 1$ *have multiplicity not exceeding the order of the differential equation,* see [6]. *Moreover, as* $h \to 0,\ \rho(z) = z^{r-\mu}(z-1)^{\mu}$ *where* μ *is the order of the differential equation,* r *is the order of the matrices* $\mathbf{A}^{(0)}$ *and* $\mathbf{E}$ *,* see [7] for details.

For our quarter-step method,

$$\rho(z) = \left| z\begin{bmatrix} 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \end{bmatrix} - \begin{bmatrix} 0&0&0&0&0&1 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&1 \end{bmatrix} \right| = 0 \quad (14)$$

$\rho(z) = z^5(z-1) = 0, \Rightarrow z_1 = z_2 = z_3 = z_4 = z_5 = 0, z_6 = 1$. Hence, the quarter-step method is zero-stable.

3.3 Consistency of the Quarter-step Method

The quarter-step method is consistent since it has order $p = 6 \ge 1$.

3.4 Convergence of the Quarter-step Method

The quarter-step method is convergent by consequence of Dahlquist theorem below.

Theorem 2 [8]: *The necessary and sufficient conditions that a continuous LMM be convergent are that it be consistent and zero-stable.*

3.5 Region of Absolute Stability of the Quarter-step Method

Definition 5 : *Region of absolute stability is a region in the complex* z *plane, where* $z = \lambda h$ *. It is defined as those values of* z *such that the numerical solutions of* $y' = -\lambda y$ *satisfy* $y_j \to 0 \ as \ j \to \infty$ *for any initial condition,* see [9].

We shall adopt the boundary locus method to determine the region of absolute stability of the quarter-step method. This is achieved by substituting the test equation,

$$y' = -\lambda y \tag{15}$$

into the block formula gives (8). This gives,

$$\mathbf{A}^{(0)}\mathbf{Y}_m(w) = \mathbf{E}\mathrm{y}_n(w) - h\lambda\mathbf{D}y_n(w) - h\lambda\mathbf{B}\mathbf{Y}_m(w) \tag{16}$$

Thus,

$$\bar{h}(w) = -\left(\frac{\mathbf{A}^{(0)}Y_m(w) - \mathbf{E}y_n(w)}{\mathbf{D}y_n(w) + \mathbf{B}Y_m(w)}\right) \tag{17}$$

since $\bar{h}$ is given by $\bar{h} = \lambda h \ \ and \ \ w = e^{i\theta}$. Equation (17) is our characteristic/stability polynomial. For the new quarter-step method, equation (17) is given by,

$$\begin{aligned} \bar{h}(w) = {} & h^6\left(\frac{16911563}{13376773560729600}w^6 - \frac{5947}{9364045824000}w^5\right) - h^5\left(\frac{96010940393}{2140283769716736000}w^6 + \frac{6233}{73156608000}w^5\right) \\ & + h^4\left(\frac{772779871}{116117826048000}w^6 - \frac{907343}{146313216000}w^5\right) - h^3\left(\frac{79135273807}{371577043353600}w^6 + \frac{3749}{15052800}w^5\right) \\ & + h^2\left(\frac{155480573}{20479334400}w^6 - \frac{17561}{2822400}w^5\right) - h\left(\frac{1}{8}w^6 + \frac{179}{1120}w^5\right) + w^6 - w^5 \end{aligned} \tag{18}$$

This gives the region of absolute stability shown in the figure below.

From Fig. 1, the RAS is L-stable because it contains the left-half of the complex plane and the stability polynomial in (18) tends to zero as $w \to \infty$. Matlab software was used to plot the RAS.

4 Numerical Experiments

We shall apply the newly developed quarter-step method on some first-order initial value problems which have appeared in literature and compare the results with solutions from some methods of similar derivation. The following notations shall be used in the tables below;

ERR - |Exact Solution-Computed Solution|
ERJ - Error in [10]

4.1 Numerical Examples

Problem 1

Consider the ODE

$$y' = x - y,\ y(0) = 0,\ 0 \le x \le 1,\ h = 0.1 \quad (19)$$

which has the exact solution,

$$y(x) = x + e^{-x} - 1 \quad (20)$$

Problem 2

Consider the ODE,

$$y' = xy,\ y(0) = 1,\ 0 \le x \le 1,\ h = 0.1 \quad (21)$$

with the exact solution,

$$y(x) = e^{\frac{1}{2}x^2} \quad (22)$$

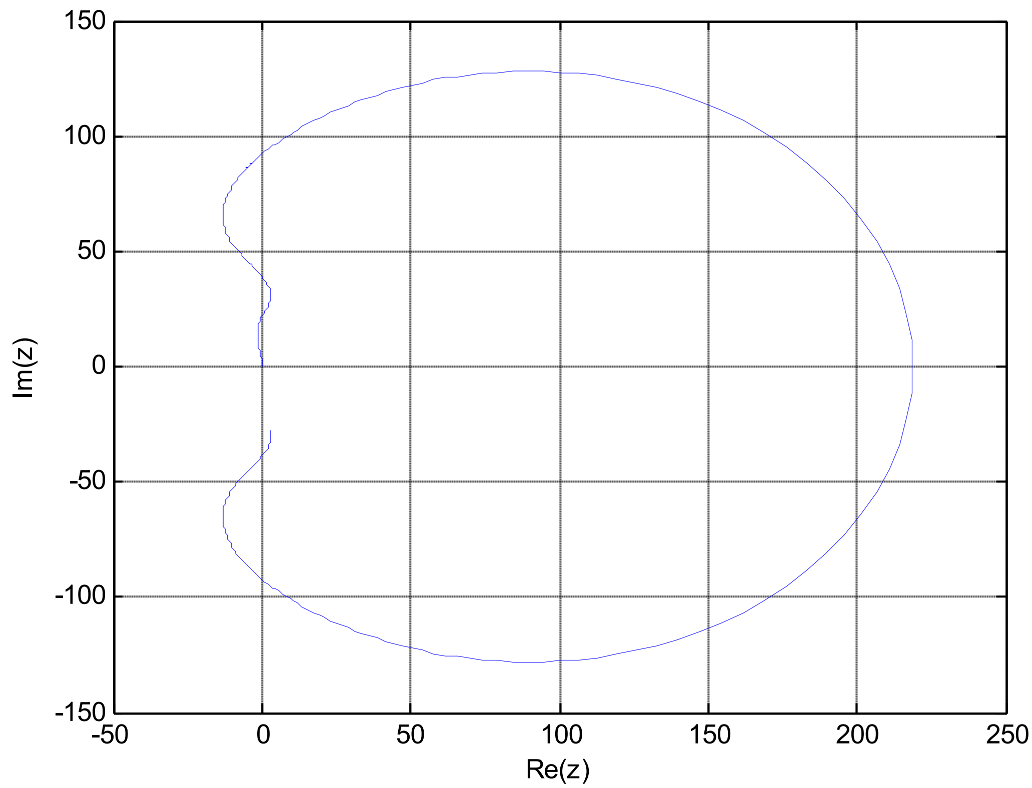

Fig. 1. Showing the region of absolute stability of the quarter-step method

Table 1. Showing the results for problem 1

x	Exact solution	Computed solution	ERR	ERJ
0.10	0.00483741803596	0.00483741803596	1.0899(-14)	1.7443(-11)
0.20	0.01873075307798	0.01873075307798	3.6577(-14)	1.5786(-11)
0.30	0.04081822068172	0.04081822068172	4.4761(-14)	1.4283(-11)
0.40	0.07032046035649	0.07032046035649	6.1209(-14)	1.2924(-11)
0.50	0.10653065971263	0.10653065971263	6.1209(-14)	1.1694(-11)
0.60	0.14881163609403	0.14881163609403	7.0592(-14)	1.0581(-11)
0.70	0.19658530379141	0.19658530379141	7.9268(-14)	9.5739(-12)
0.80	0.24932896411722	0.24932896411722	8.3601(-15)	8.6613(-12)
0.90	0.30656965974060	0.30656965974060	9.4146(-15)	7.8396(-12)
1.00	0.36787944117144	0.36787944117144	9.7071(-15)	7.0906(-12)

Table 2. Showing the results for problem 2

x	Exact solution	Computed solution	ERR	ERJ
0.10	1.005012520887401	1.005012520887400	1.2473(-13)	1.6554(-11)
0.20	1.020201340026755	1.020201340026753	2.4989(-13)	4.3981(-11)
0.30	1.046027859908716	1.046027859908711	4.0149(-13)	7.8451(-11)
0.40	1.083287067674958	1.083287067674951	5.7196(-13)	1.2662(-11)
0.50	1.133148453066826	1.133148453066819	7.5116(-13)	1.9709(-10)
0.60	1.197217363131810	1.197217363131801	9.2698(-13)	3.0180(-10)
0.70	1.277621313204886	1.277621313204855	3.0572(-12)	4.5771(-10)
0.80	1.377129776433595	1.377129776433564	3.1135(-12)	6.8954(-09)
0.90	1.499302500056767	1.499302500056705	6.1995(-12)	1.0336(-09)
1.00	1.644872127070013	1.644872127069923	6.6348(-12)	1.5435(-09)

4.2 Discussion of Results

We considered two numerical examples in this paper. The two problems were earlier solved by the authors in [10], where they applied an order seven hybrid block method. We applied a new order six quarter-step hybrid block method to solve these two problems and from the results obtained, the quarter-step method performed better than the existing method with which we compared our results. It was also observed that our uniform order six method performed better than the order seven method developed by authors in [10].

5 Conclusion

We have developed a new method called a quarter-step method for the solution of first-order ordinary differential equations. The method was applied on some problems and from the results obtained it shows that the method is more computationally reliable than the existing one. The method was also found to be zero-stable, consistent and convergent. This method is therefore recommended for the solution of problems of the form (1).

Competing Interests

Authors have declared that no competing interests exist.

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