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Interplanetary Relationship with Some Parameters of Two Planets Based on the Angular Diameter of the Sun

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

This paper describes an interplanetary relationship between any two planets. This relation is between the distances of planets from the Sun, axis tilts of the planets, the diameter of planets and equatorial rotational velocities of planets. This one elegant equation sums most of the physical quantities related to a planet. The expression uses the concept that the angular diameter of the Sun is different from each planet. On rearranging and substituting different values in the expression we can expand the expression's functionality by including the mass and densities of planets. In the interplanetary relationship expression we will see two cases; one, the special case which gives an expression which shows the relation between Earth and all of the planets of the solar system. Two, the general case which gives an expression (which expands the application of the expression) using which we can form a relationship between any two planets of the solar system. If certain rules are followed then we get a relation which is dimensionally and numerically equal. On rearranging the terms of the interplanetary relationship expression, we get two new expressions for finding the angular diameter of the sun from different planets. Using the equations of angular diameter we

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derive an interplanetary relationship expression independent of angular diameter of the sun. Using the expressions for the angular diameter of the sun we will also find the value of a constant which represents the ratio of the physical characteristics of a planet. We will verify and understand the application of all these equations by substituting real values for each planet.

Keywords: Interplanetary relationship; angular diameter; axis tilt; diameter; distance.

1. INTRODUCTION

In our universe nothing is independent; everything is interconnected with each other through mathematics. Sometimes mathematics binds such quantities that don't seem related to each other directly; this is the beauty of mathematics. This is the case with this paper; in this paper, we will see relations between quantities which seem to be independent of each other.

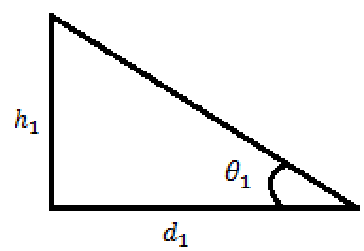
Before we talk about the interplanetary relationship expression we will talk about a previous paper of mine from which I got the idea for the interplanetary relationship expression (Skip to page 3 for the interplanetary relationship). In a paper with the title "Expression for the relative change of height when the distance between the viewer and the body changes; and its cosmological applications."[1], I discussed about the change in the apparent height of a body in the frame of reference of the viewer when the distance between the body and the viewer changes. I termed this apparent height, which was relative to the viewer, as relative height.

In that paper, I had derived an expression with which we could calculate this relative height; for deriving this expression I had developed a thought experiment. The thought experiment was as follows: Imagine you are looking at a building; then you slowly start moving away from it and stop after some distance. Now notice the appearance of the building; how does it look? Also, note what happens to the angle of elevation as you move away from the building.

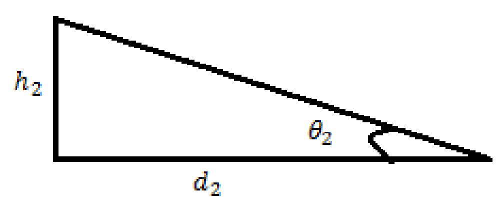
So what we can see is that as we move away from the building; the building appears smaller and smaller as the distance increases. Also as you move away from the building, you can notice that the pitch of your head decreases. This means that the angle of elevation to the top of the building is decreasing as you move away from it. This thought experiment gave us 2 quantities on which relative height is dependent: one, the distance between the body and the viewer, which is inversely proportional. Two, the angle of elevation, which is directly proportional. The third quantity which affects the relative height is the actual height of the building, which is directly proportional. For this one, imagine you are looking at a tall pole and man both from the same distance. Which appears to be bigger? The pole appears to be bigger as its actual height is more than that of a man.

Consider the following diagrams for a better understanding.

Initial case (when we are standing close to the building):



Final case (when we have moved away from the building):



This is the diagram which shows our thought experiment. As we can see that as distance increases $(d_1 < d_2)$ and the angle of elevation decreases $(\theta_2 < \theta_1)$ hence, as a result, the relative height decreases $(h_2 < h_1)$

Hence we get this relation,

$$relative\ height \propto \theta$$

$$relative\ height \propto \frac{1}{d}$$

$$relative\ height \propto \mathrm{h}$$

$$\therefore \quad \Delta h \propto \frac{\theta h}{d}$$

On removing the proportionality we get,

$$\Delta h = k\frac{\theta h}{d} \quad (1)$$

Where k is the proportionality constant.

Here,
Δh = relative height
θ = angle of depression / elevation in radian
d = distance between the body and the viewer
h = actual height of the body

To prove the validity of the equation 1, I considered the fact that the size of the sun appears different from each planet and as the distance from the sun increases the size of the sun appears to decrease. This means that the sun appears the biggest from Mercury and as we on to the other planets the size of the sun appears to decreases. Hence I considered the size of the sun relative to a planet as the relative height of the sun from that planet.

Also to my advantage, previous studies had given me the approximate size of the sun from different planets in comparison with the size of sun seen from planet Earth. For example, the size of the sun from Mars appears to be $^2/_3$ [2] times the size of the sun that appears from Earth. So if we consider the size of the sun that appears from Earth to be X, then the size of the sun from Mars would appear $\frac{2}{3}X$. This X is the relative height of the sun from Earth and $\frac{2}{3}X$ is the relative height of the sun from Mars. The idea was to form two relative height equations using equation 1, one for Earth and second for Mars. We get the following,

The expression for relative height of the sun in the frame reference of Earth using equation 1 is

$$X = \frac{\theta h}{d}k \quad (2)$$

Where,
X = Relative height of the Sun from Earth
θ = Axis tilt of Earth in radians
h= Diameter of the Sun
d = Distance between Earth and Sun

The expression for relative height of the sun in the frame reference of Mars using equation 1 is

$$\frac{2}{3}X \quad = \frac{\emptyset h}{D}k \quad (3)$$

Where,

X = Relative height of the Sun from Earth
∅ = Axis tilt of Mars
h = Diameter of the Sun
D= Distance between Mars and Sun
$\frac{2}{3}X$ = Relative height of the Sun from Mars

In the next step I substituted the value of X from equation 2 in equation 3, giving the expression:

$$\frac{2}{3}\left(\frac{\theta h}{d}k\right) = \frac{\emptyset h}{D}k$$

Cancelling out the common terms (diameter of Sun is constant (h) and the proportionality constant k) we get,

$$\frac{2}{3}\left(\frac{\theta}{d}\right) = \frac{\emptyset}{D} \quad (4)$$

Actual values for each quantity
For Earth [3],
θ= 0.410 rad
d = $149.6 * 10^9\ m$

For Mars [4],
∅ = 0.436 rad
D = $227.9 * 10^9\ m$

Substituting these values in equation 4

$$\frac{2}{3}\left(\frac{0.410}{149.6 * 10^9}\right) = \frac{0.436}{227.9 * 10^9}$$

LHS = $1.82709 * 10^{-12}$
RHS = $1.91312 * 10^{-12}$

As we can see RHS and LHS are nearly equal. The RHS and LHS of equation 4 being nearly equal made me think that equation 1 which we derived for relative height was true. But that was not the case. Like the way we found the relation for Mars and Earth, I tried substituting values for other planets in equation 4; but it didn't work properly for other planets and gave undesired results. Also, I noticed taking the axis tilts of planets as the angle of the elevation was wrong because the angle of elevation was directly proportional to relative height and angle of elevation decreased as the distance increased. But the axis tilt of Mars is more than that of Earth even when the distance between Mars and the sun is more than that of Earth and the sun; also axis tilts of planets were independent of distance from the sun. Hence this proof which we performed in equation 4 couldn't be used to prove equation 1. Also on working more on equation 1, I realized that the concept relative height was wrong and didn't have any physical relevance. The relative height I was talking about was very similar to angular diameter. It was the angular diameter which decreased and increased depending on the distance and hence there was no need of having the relative height concept. Also while working on some real-life applications of equation 1 I got unrealistic results. Also, it is an optical illusion that the height of objects appears to be different from different points; hence the concept of relative height is only an illusion. This means that equation 1 is incorrect and incomplete; the concept of relative height is wrong.

But equation 4 got my attention and made me think why I was getting this relation true for Earth and Mars. So instead of working on an equation on relative height, I started working on an interplanetary relationship like equation 4. Working on an expression like equation 4, I obtained an interplanetary relationship which covered all planets of the solar system. With many thought experiments and trial and error method, I obtained this expression. We will further talk about two cases of the interplanetary relationship expression; one is a special case in which we compare all planets to Earth. Second is the general case in which a relationship can be formed between any two planets of the solar system.

2. THE SPECIAL CASE OF THE INTERPLANETARY RELATIONSHIP EXPRESSION

For understanding the equation, we need to perform this thought experiment:

Imagine, we are on Earth, and from Earth, we observe the Sun and also we note the angular diameter of the Sun from Earth. Also, we find out the distance between Earth and Sun to be "d", the axis tilt of Earth to be θ, the equatorial rotational velocity of Earth to be V_A and diameter of Earth to be "h". Keeping in mind all the values we travel to planet B.

Now from Planet B we observe the Sun and find out that the angular diameter of the Sun from planet B is "z" times the angular diameter of the sun we saw from Earth. Also, we find out the distance between Sun and planet B to be "D", the equatorial rotational velocity of planet B to be V_B, the axis tilt of planet B to be Ø and the diameter of planet B to be "H". Using all these variables we will from the special case of interplanetary relationship.

The expression for the special case of interplanetary relationship is:

$$\boldsymbol{z}\left(\frac{\boldsymbol{\theta V_A}}{\boldsymbol{d^3\,h}}\right) = \frac{\boldsymbol{\emptyset V_B}}{\boldsymbol{D^2 H}} \qquad (5)$$

Where,
On the LHS (Earth)

$$z = \frac{Angular\ diameter\ of\ Sun\ from\ planet\ B}{Angular\ diameter\ of\ Sun\ from\ Earth} = \frac{\delta_B}{\delta_A}$$

θ = Axis tilt of Earth (radians)
V_A= Equatorial rotational velocity of Earth (m/s)
d = Distance between Sun and Earth (m)
h = Diameter of Earth (m)
δ_A = Angular diameter of the sun from Earth (radians)

On the RHS (Planet B)
Ø = Axis tilt of planet B (radian)
V_B= Equatorial rotational velocity of Planet B (m/s)
D = Distance between Sun and Planet B (m)
H = Diameter of Planet B (m)
δ_B = Angular diameter of the sun from Planet B (radians)

$$\underbrace{\boldsymbol{z}\left(\frac{\boldsymbol{\theta V_A}}{\boldsymbol{d^3\,h}}\right)}_{\substack{\text{Represents} \\ \text{Earth} \\ \text{(LHS)}}} = \overbrace{\frac{\boldsymbol{\emptyset V_B}}{\boldsymbol{D^2 H}}}^{\substack{\text{(RHS)} \\ \text{Represents} \\ \text{Planet B}}}$$

3. DIMENSIONAL ANALYSES OF THE EXPRESSION

On substituting the units of each quantity in the equation 5 we get,

$$\frac{rad}{rad}\left(\frac{rad*ms^{-1}}{m^3\,m}\right) = \frac{rad*ms^{-1}}{m^2\,m}$$

Cancelling out the common units gives

$$m^{-1} = 1$$

As we can see the LHS and RHS are not balanced dimensionally. Also when we see the application of the expression we will see that the powers of LHS and RHS are not balanced.

4. THE SOLUTION TO DIMENSIONAL INEQUALITY AND UNBALANCED POWERS

As we will see in the upcoming examples that the powers of RHS and LHS are not balanced. To balance the powers there is a pattern. And we will take advantage of this pattern to make RHS and LHS equal. This pattern is dependent on a range; the position of a certain planet in this range determines the value multiplied to the RHS of equation 5. The position of the planet in this range is determined by the ratio of that planet's diameter and the distance between that planet and the Sun. This ratio is constant and is represented by the alphabet T.

$$T = \frac{H}{D} = \frac{Diameter\ of\ planet\ B}{Distance\ between\ sun\ and\ planet\ B}$$

Planet B is the planet which is being taken on the RHS.

The range and its rules are:

1. If the value of T for a planet is greater than $1.5 * 10^{-5}$ and less than $8.517 * 10^{-5}$ then the RHS will be multiplied by $10^{-11}\ m^{-1}$

2. If the value of T for a planet does not lie between $1.5 * 10^{-5}$ and $8.517 * 10^{-5}$ then RHS will be multiplied by $10^{-10}\ m^{-1}$

3. If the value of T for a is greater than $8.517 * 10^{-5}$ and less than $1 * 10^{-4}$ then RHS will be multiplied by $10^{-7}\ m^{-1}$

4. If the value of T for a planet is equal to $8.517 * 10^{-5}$ (for Earth) then the RHS is multiplied by $10^{-11}\ m^{-1}$

By following these rules we can get numerically and dimensionally equal LHS and RHS. This range is chosen because it accommodates and satisfies the expression for each planet. This range is chosen by trial and error method. The range in itself has no significance; the range and its rules only serve the purpose of mathematically aligning the results for all the planets of our solar system.

Currently, I have no derivation for this expression, but we can prove its validity by substituting real values in the expression.

5. PROOFS AND EXAMPLES OF THE SPECIAL CASE OF INTERPLANETARY RELATIONSHIP EXPRESSION

In all cases we will compare the size of the Sun that appears from Earth with the size of the sun as seen from rest of the planets of the solar system; hence it's the special case.

Table 1. This table shows the angular diameter of the sun from each planet

Name of planet	The angular diameter of the Sun [5]	
Mercury	1.4°	(0.024 rad)
Venus	0.74°	(0.013 rad)
Earth	0.53°	(0.0093 rad)
Mars	0.35°	(0.0061 rad)
Jupiter	0.10°	(0.0018 rad)
Saturn	0.056°	(0.00097 rad)
Uranus	0.028°	(0.00048 rad)
Neptune	0.018°	(0.00031 rad)

5.1 Example: Relation between Earth and Mars

The size of the Sun from Mars appears to be 0.65 times the size of the Sun that appears from Earth. Hence Mars will be on LHS and Earth on the RHS.

For Earth [3],
$\theta = 0.410\ rad$
$V_A = 465.1\ m/s$
$d = 149.6 * 10^9\ m$
$h = 12742 * 10^3\ m$
$\delta_A = 0.0093\ rad$

For Mars (Planet B) [4],
$\emptyset = 0.436\ rad$
$V_B = 241.7\ m/s$
$D = 227.9 * 10^9\ m$
$H = 6779 * 10^3\ m$
$\delta_B = 0.0061\ rad$

T for Mars,

$$T = \frac{H}{D} = \frac{6779 * 10^3}{227.9 * 10^9} = 2.97455 * 10^{-5}$$

For z,

$$z = \frac{Angular\ diameter\ of\ Sun\ from\ planet\ B}{Angular\ diameter\ of\ Sun\ from\ Earth}$$

Referring to Table 1 for values

$$z = \frac{0.0061\ rad}{0.0093\ rad} = 0.65$$

Substituting the values in equation 5

LHS =
$$z\left(\frac{0.410 * 465.1}{(149.6 * 10^9)^3 \ * 12742 * 10^3}\right)\frac{rad^1}{m^3 s^1} \quad (6)$$

On substituting the value of z in equation 6 gives us,

LHS =
$$0.65\left(\frac{0.410 * 465.1}{(149.6 * 10^9)^3 \ * 12742 * 10^3}\right) rad^1 m^{-3} s^{-1}$$

RHS =
$$\left(\frac{0.436 * 241.7}{(227.9 * 10^9)^2 * 6779 * 10^3}\right) rad^1 m^{-2} s^{-1}$$

Simplifying the LHS and RHS by cancelling out common terms and units

LHS =
$$0.65\left(\frac{0.410 * 465.1}{(149.6)^3 \ * 12742 * 10^9}\right) m^{-1}$$

RHS =
$$\left(\frac{0.436 * 241.7}{(227.9)^2 * 6779}\right)$$

LHS = $2.90 * 10^{-18}\, m^{-1}$
RHS = $2.99 * 10^{-7}$

As mentioned before that the RHS and LHS are not balanced dimensionally and powers are not balanced, we can see that in the above example.

As T for Mars lies between $1.5 * 10^{-5}$ and $8.517 * 10^{-5}$ therefore because of rule number 1, RHS will be multiplied with $10^{-11}\, m^{-1}$.

$$\therefore RHS = 2.99 * 10^{-18}\, m^{-1}$$

And therefore RHS and LHS are nearly equal.

5.2 Example: Relationship between Earth and Jupiter

The size of the sun from Jupiter appears to be 0.1935 times the size of the sun that appears from Earth. By changing the value of z in equation 6 we can obtain the LHS for example 2. For Jupiter (Planet B) [6]

$\emptyset = 0.05462\, rad$
$V_B = 12600\, m/s$
$D = 776.82 * 10^9\, m$
$H = 139820 * 10^3\, m$
$\delta_B = 0.0018\, rad$

For z,
$$z = \frac{Angular\ diameter\ of\ Sun\ from\ planet\ B}{Angular\ diameter\ of\ Sun\ from\ Earth}$$

Referring to Table 1 for values,

$$z = \frac{0.0018\, rad}{0.0093\, rad} = 0.1935$$

T for Jupiter,

$$T = \frac{H}{D} = \frac{139820 * 10^3}{776.82 * 10^9} = 1.79964 * 10^{-4}$$

By substituting the values in equation 5 we get,

LHS=
$$0.1935\left(\frac{0.410 * 465.1}{(149.6 * 10^9)^3 \ * 12742 * 10^3}\right) rad^1 m^{-3} s^{-1}$$

RHS =
$$\left(\frac{0.054 * 12600}{(776.82 * 10^9)^2 * 139820 * 10^3}\right) rad^1 m^{-2} s^{-1}$$

Simplifying the LHS and RHS by cancelling out common terms and units

LHS =
$$0.1935\left(\frac{0.410 * 465.1}{(149.6)^3 \ * 12742 * 10^9}\right) m^{-1}$$

RHS =
$$\left(\frac{0.05462 * 12600}{(776.82)^2 * 139820}\right)$$

LHS = $8.649 * 10^{-19}\, m^{-1}$
RHS = $8.1566 * 10^{-9}$

As T for Jupiter doesn't lie between $1.5 * 10^{-5}$ and $8.517 * 10^{-5}$ therefore because of rule number 2, RHS will be multiplied with $10^{-10}\, m^{-1}$.

∴ RHS =$8.1566 * 10^{-19}\, m^{-1}$

Therefore RHS and LHS are nearly equal.

5.3 Example: Relationship between Earth and Neptune

The size of the sun from Neptune appears to be 0.03times the size of the sun that appears from Earth. By changing the value of z in equation 6 we can obtain the LHS for example 3.

For Neptune (Planet B) [7],
$\emptyset = 0.4942\ rad$
$V_B = 2680\ m/s$
$D = 4.495 * 10^{12}\ m$
$H = 49244 * 10^3\ m$
$\delta_B = 0.00031\ rad$

For z,

$$z = \frac{Angular\ diameter\ of\ Sun\ from\ planet\ B}{Angular\ diameter\ of\ Sun\ from\ Earth}$$

Referring to Table 1 for values,

$$z = \frac{0.00031\ rad}{0.0093\ rad} = 0.03$$

T for Neptune,

$$T = \frac{H}{D} = \frac{49244 * 10^3}{4.495 * 10^{12}} = 1.09 * 10^{-5}$$

By substituting the values in equation 5 we get,

LHS=
$$0.03\left(\frac{0.410 * 465.1}{(149.6 * 10^9)^3\ * 12742 * 10^3}\right) rad^1 m^{-3} s^{-1}$$

RHS =
$$\left(\frac{0.494 * 2680}{(4.495 * 10^{12})^2 * 49244 * 10^3}\right)\ rad^1 m^{-2} s^{-1}$$

Simplifying the LHS and RHS by cancelling out common terms and units

LHS =
$$0.03\left(\frac{0.410 * 465.1}{(149.6)^3\ * 12742 * 10^3}\right) m^{-1}$$
RHS =
$$\left(\frac{0.494 * 2680}{(4.495)^2 * 49244}\right)$$

LHS = $1.3409 * 10^{-13}\ m^{-1}$
RHS = $1.3306 * 10^{-3}$

As T for Neptune doesn't lie between $1.5 * 10^{-5}$ and $8.517 * 10^{-5}$ therefore because of rule number 2, RHS will be multiplied with $10^{-10}\ m^{-1}$.

∴RHS = $1.3306 * 10^{-13}\ m^{-1}$

Therefore RHS and LHS are nearly equal.

5.4 Example: Relationship between Earth and Venus

The size of the sun from Venus appears to be 1.397 times the size of the sun that appears from Earth. By changing the value of z in equation 6 we can obtain the LHS for example 4.

For Venus (Planet B) [8],
$\emptyset = 3.09\ rad$
$V_B = 1.81\ m/s$
$D = 108.03 * 10^9\ m$
$H = 12104 * 10^3\ m$
$\delta_B = 0.013\ rad$

For z,
$$z = \frac{Angular\ diameter\ of\ Sun\ from\ planet\ B}{Angular\ diameter\ of\ Sun\ from\ Earth}$$

Referring to Table 1 for values,

$$z = \frac{0.013\ rad}{0.0093\ rad} = 1.397$$

T for Venus,

$$T = \frac{H}{D} = \frac{12104 * 10^3}{108.03 * 10^9} = 1.12 * 10^{-4}$$

By substituting the values in equation 5 we get,

LHS=
$$1.397\left(\frac{0.410 * 465.1}{(149.6 * 10^9)^3\ * 12742 * 10^3}\right) rad^1 m^{-3} s^{-1}$$

RHS =
$$\left(\frac{3.09 * 1.81}{(108.03 * 10^9)^2 * 12104 * 10^3}\right)\ rad^1 m^{-2} s^{-1}$$

Simplifying the LHS and RHS by cancelling out common terms and units

LHS =
$$1.397\left(\frac{0.410 * 465.1}{(149.6)^3\ * 12742 * 10^9}\right) m^{-1}$$

RHS =
$$\left(\frac{3.09 * 1.81}{(108.03)^2 * 12104}\right)$$

LHS = $6.63571 * 10^{-18}\ m^{-1}$
RHS = $3.95931 * 10^{-8}$
As T for Venus doesn't lie between $1.5 * 10^{-5}$ and $8.517 * 10^{-5}$ therefore because of rule number 2, RHS will be multiplied with $10^{-10}\ m^{-1}$.

∴RHS = $3.94757 * 10^{-18}\ m^{-1}$
Therefore RHS and LHS are nearly equal.

5.5 Example: Relationship between Earth and Mercury

The size of the sun from Mercury appears to be 2.5806 times the size of the sun that appears from Earth. By changing the value of z in equation 6 we can obtain the LHS for example 5.

For Mercury (Planet B) [9],
$\emptyset = 0.0005934\, rad$
$V_B = 3.026\, m/s$
$D = 50.726 * 10^9\, m$
$H = 4880 * 10^3\, m$
$\delta_B = 0.024\, rad$

For z,

$$z = \frac{Angular\ diameter\ of\ Sun\ from\ planet\ B}{Angular\ diameter\ of\ Sun\ from\ Earth}$$

Referring to Table 1 for values,

$$z = \frac{0.024\, rad}{0.0093\, rad} = 2.5806$$

T for Mercury,

$$T = \frac{H}{D} = \frac{4880 * 10^3}{50.726 * 10^9} = 9.62031 * 10^{-5}$$

By substituting the values in equation 5 we get,

LHS=
$$\left(\frac{2.5806 * 0.410 * 465.1}{(149.6 * 10^9)^3\ * 12742 * 10^3}\right) rad^1 m^{-3} s^{-1}$$

RHS =
$$\left(\frac{0.0005934\ * 3.026}{(50.726 * 10^9)^2 * 4880 * 10^3}\right)\ rad^1 m^{-2} s^{-1}$$

Simplifying the LHS and RHS by cancelling out common terms and units

LHS =
$$2.5806 \left(\frac{0.410 * 465.1}{(149.6)^3\ * 12742 * 10^9}\right) m^{-1}$$

RHS =
$$\left(\frac{0.0005934\ * 3.026}{(50.726)^2 * 4880}\right)$$

LHS = $1.15 * 10^{-17}\, m^{-1}$
RHS = $1.427 * 10^{-10}$

As T for Mercury lies between $8.517 * 10^{-5}$ and $1 * 10^{-4}$ therefore because of rule number 3,

RHS will be multiplied with $10^{-7}\, m^{-1}$.

∴RHS = $1.42797 * 10^{-17}\, m^{-1}$

Therefore RHS and LHS are nearly equal.

All these examples are also the proofs of the interplanetary expression. In all of the cases, we got LHS and RHS nearly equal. We didn't get RHS and LHS exactly because the values of diameter, distance, axis tilt and equatorial rotational velocities of planets are not exact.

In the above examples, we compared the size of the sun as seen from different planets to the size of the sun seen from Earth. It was a special case where we considered a relationship between only Earth and other planets. But a general case would include a relationship between any 2 planets of the solar system.

6. THE GENERAL CASE OF INTERPLANETARY RELATIONSHIP

We need an expression, which can include the results of the special case without any change. For the general case, we needed some quantity which is unique to a planet and the value of that quantity should be 1 for Earth. The only possible quantity which could satisfy both the conditions is the number of moons. Now the question we face is whether we take the number moons of the planet on LHS or the planet which is on the RHS? If we take the number of moons of the planet B and multiply it on RHS itself, then we won't be able to satisfy the special case of Earth. Hence we will take the number of moons of planet A and multiply that value to LHS itself.

We get the expression,

$$\boldsymbol{zn}\left(\frac{\boldsymbol{\theta V_A}}{\boldsymbol{d^3\, h}}\right) = \frac{\boldsymbol{\emptyset V_B}}{\boldsymbol{D^2 H}} \tag{7}$$

Where,

$$z = \frac{Angular\ diameter\ of\ Sun\ from\ planet\ B}{Angular\ diameter\ of\ Sun\ from\ planet\ A} = \frac{\delta_A}{\delta_B}$$

θ = Axis tilt of Planet A (radians)
V_A= Equatorial rotational velocity of Planet A (m/s)
d = Distance between Sun and Planet A (m)
h = Diameter of Planet A (m)
n = Number of moons of Planet A (unit less)

$\emptyset$ = Axis tilt of planet B (radian)
V_B= Equatorial rotational velocity of Planet B (m/s)
D = Distance between Sun and Planet B (m)
H = Diameter of Planet B (m)
The rules which we followed for the special case will be the same for the general case also.

$$\underbrace{zn\left(\frac{\theta V_A}{d^3\,h}\right)}_{\substack{\text{LHS}\\ \text{(Represents Planet A)}}} = \overbrace{\frac{\emptyset V_B}{D^2 H}}^{\substack{\text{RHS}\\ \text{(Reprsents Planet B)}}}$$

Repeating the rules and range for T, where T is,

$$\mathrm{T} = \frac{H}{D} = \frac{\textit{Diameter of planet B}}{\textit{Distance between sun and planet B}}$$

The range and its rules:

1. If the value of T for a planet is greater than $1.5 * 10^{-5}$ and less than $8.517 * 10^{-5}$ then the RHS will be multiplied by $10^{-11}\ m^{-1}$
2. If the value of T for a planet does not lie between $1.5 * 10^{-5}$ and $8.517 * 10^{-5}$ then RHS will be multiplied by $10^{-10}\ m^{-1}$
3. If the value of T for a is greater than $8.517 * 10^{-5}$ and less than $1 * 10^{-4}$ then RHS will be multiplied by $10^{-7}\ m^{-1}$
4. If the value of T for a planet is equal to $8.517 * 10^{-5}$ (for Earth) then the RHS is multiplied by $10^{-11}\ m^{-1}$

The general case expression does not work for planets with zero moons (Mercury and Venus). For finding quantities related to such planets we will take them on the RHS, as on the RHS the number of moons is not considered.

Everything else remains the same as the general case. Again, I don't have the derivation for this expression. So we will give proof of the equation by substituting real values. We will refer Table 1 for all the values of angular diameter.

6.1 Example: Relationship between Jupiter and Neptune

In this case, we will be comparing the size of the sun as seen from Neptune with the size of the sun of as seen from Jupiter.

For Jupiter (Planet A) [6],
$\theta = 0.05462\ rad$
$V_A = 12600\ m/s$
$d = 776.82 * 10^9\ m$
$h = 139820 * 10^3\ m$
n = 79
$\delta_A = 0.0018\ rad$

For Neptune (Planet B) [7],
$\emptyset = 0.4942\ rad$
$V_B = 2680\ m/s$
$D = 4.495 * 10^{12}\ m$
$H = 49244 * 10^3\ m$
$\delta_B = 0.00031\ rad$

T for Neptune,

$$T = \frac{H}{D} = \frac{49244 * 10^3}{4.495 * 10^{12}} = 1.09 * 10^{-5}$$

For z,

$$z = \frac{\delta_A}{\delta_B} = \frac{0.00031}{0.0018} = 0.172$$

By substituting the values in equation 7,

LHS =
$$\left(\frac{79 * 0.172 * 0.054 * 12600}{(776.82 * 10^9)^3 * 139820 * 10^3}\right)\frac{rad}{m^3 s^1}$$

RHS =
$$\left(\frac{0.494 * 2680}{(4.495 * 10^{12})^2 * 49244 * 10^3}\right)\ rad^1 m^{-2} s^{-1}$$

Simplifying the equation by cancelling out the common numbers and units

LHS =
$$\left(\frac{79 * 0.172 * 0.054 * 12600}{(776.82)^3 * 139820 * 10^3}\right)\ m^{-1}$$

RHS =
$$\left(\frac{0.494 * 2680}{(4.495)^2 * 49244}\right)$$

LHS = $1.42675 * 10^{-13}\ m^{-1}$
RHS = $1.3314 * 10^{-3}$

As T for Neptune doesn't lie between the range $1.5 * 10^{-5}$ and $8.517 * 10^{-5}$, hence RHS will be multiplied by $10^{-10}\ m^{-1}$

∴RHS = $1.3314 * 10^{-13}\ m^{-1}$

Therefore RHS and LHS are nearly equal.

6.2 Example: Relationship between Mars and Mercury

In this case, we will compare the size of the sun as seen from Mercury with the size of the sun as seen from Mars.

For Mars (Planet A) [4],
$\theta = 0.436\, rad$
$V_A = 241.7\, m/s$
$d = 227.9 * 10^9\, m$
$h = 6779 * 10^3\, m$
n = 2
$\delta_A = \ 0.0061\, rad$

For Mercury (Planet B) [9],
$\emptyset = 0.0005934\, rad$
$V_B = 3.026\, m/s$
$D = 50.726 * 10^9\, m$
$H = 4880 * 10^3\, m$
$\delta_B = 0.024\, rad$

T for Mercury,

$$T = \frac{H}{D} = \frac{4880 * 10^3}{50.726 * 10^9} = 9.62031 * 10^{-5}$$

For z,

$$z = \frac{\delta_A}{\delta_B} = \frac{0.024}{0.0061} = 3.93$$

By substituting the values in equation 7,

LHS =
$$\left(\frac{3.93443 * 2 * 0.436 * 241.7}{(227.9 * 10^9)^3 * 6779 * 10^3}\right)\ rad^1 m^{-3} s^{-1}$$

RHS =
$$\left(\frac{0.0005934 * 3.026}{(50.762 * 10^9)^2 * 4880 * 10^3}\right)\ rad^1 m^{-2} s^{-1}$$

Simplifying the equation by cancelling out the common numbers and units

LHS =
$$\left(\frac{2 * 3.93 * 0.436 * 241.7}{(227.9)^3 * 6779 * 10^9}\right)\ m^{-1}$$

RHS =
$$\left(\frac{0.0005934 * 3.026}{(50.762)^2 * 4880}\right)$$

LHS = $1.03342 * 10^{-17}\, m^{-1}$
RHS = $1.43034 * 10^{-10}$

As T for Mercury lies between the range $8.517 * 10^{-5}$ and $1 * 10^{-4}$, hence RHS will be multiplied by $10^{-7}\, m^{-1}$

∴RHS = $1.43034 * 10^{-17}\, m^{-1}$

Therefore RHS and LHS are nearly equal.

6.3 Example: Relationship between Jupiter and Earth

In this case, we will compare the size of the sun as seen from Earth with the size of the sun as seen from Jupiter.

For Jupiter (Planet A) [6],
$\theta = 0.05462\, rad$
$V_A = 12600\, m/s$
$d = 776.82 * 10^9\, m$
$h = 139820 * 10^3\, m$
n = 79
$\delta_A = \ 0.0018\, rad$

For Earth (Planet B) [3],
$\emptyset = 0.410\, rad$
$V_B = 465.1\, m/s$
$D = 149.6 * 10^9\, m$
$H = 12742 * 10^3\, m$
$\delta_B = 0.0093\, rad$

T for Earth,

$$T = \frac{H}{D} = \frac{12742 * 10^3}{149.6 * 10^9} = 8.516 * 10^{-5}$$

For z,

$$z = \frac{\delta_A}{\delta_B} = \frac{0.0093}{0.0018} = 5.16$$

By substituting the values in equation 7,

LHS =
$$\left(\frac{79 * 5.16 * 0.054 * 12600}{(776.82 * 10^9)^3 * 139820 * 10^3}\right) \frac{rad}{m^3 s^1}$$

RHS =
$$\left(\frac{0.410 * 465.1}{(149.6 * 10^9)^2 * 12742 * 10^3}\right)\ rad^1 m^{-2} s^{-1}$$

Simplifying the equation by cancelling out the common numbers and units

LHS =
$$\left(\frac{79 * 5.16 * 0.054 * 12600}{(776.82)^3 * 139820 * 10^9}\right)\ m^{-1}$$

RHS =
$$\left(\frac{0.410 * 465.1}{(149.6)^2 * 12742}\right)$$

LHS = $4.28024 * 10^{-18}\, m^{-1}$
RHS = $6.68697 * 10^{-7}$

As the value of T for Earth is equal to $8.517 * 10^{-5}$ hence the RHS will be multiplied with $10^{-11}\, m^{-1}$
∴RHS = $6.68697 * 10^{-18}\, m^{-1}$

Therefore RHS and LHS are nearly equal

Considering all the examples we can see the validity of the general case expression of the interplanetary relationship.

7. INTRODUCTION OF DENSITY AND MASS OF PLANETS IN THE GENERAL CASE OF INTERPLANETARY RELATIONSHIP EXPRESSION

To increase the application of the general expression of the interplanetary relationship, we will substitute density and mass of planets in the general expression. For doing this we will find the relation between the radius of a planet, density and mass using the expression of the volume of a planet.

For Planet A, $\rho_A = density\ of\ planet\ A$ and $m_A = mass\ of\ planet\ A$

For Planet B, $\rho_B = density\ of\ planet\ B$ and $m_B = mass\ of\ planet\ B$

We know,

$$\rho_A = \frac{mass\ of\ planet\ A}{volume\ of\ planet\ A}$$

The volume of planet A (sphere) = $\frac{4}{3}\pi r^3$

The radius of planet A is $\frac{h}{2}$ because the diameter of the planet A is h.

∴ The volume of planet A = $\frac{4}{3}\pi\left(\frac{h}{2}\right)^3$

$$= \frac{4}{3}\pi\frac{h^3}{8}$$

∴ The volume of planet A = $\frac{1}{6}\pi h^3$

Substituting the value of the volume of Planet A in the expression of density of planet A

$$\rho_A = \frac{m_A}{\frac{1}{6}\pi h^3}$$

Re-writing this expression in terms of h,

$$h = \sqrt[3]{\frac{6m_A}{\pi\rho_A}}$$

Similarly,

volume for planet B = $\frac{1}{6}\pi H^3$

density of planet B = $\rho_B = \frac{m_B}{\frac{1}{6}\pi H^3}$

Re-writing this expression in terms of H,

$$H = \sqrt[3]{\frac{6m_B}{\pi\rho_B}}$$

Substituting the new values of h and H in the general expression of the interplanetary relationship that is equation 7, we get:

$$zn\left(\frac{\theta V_A}{d^3\sqrt[3]{\frac{6m_A}{\pi\rho_A}}}\right) = \frac{\emptyset V_B}{D^2\sqrt[3]{\frac{6m_B}{\pi\rho_B}}}$$

Cancelling out the common terms, we get

$$zn\left(\frac{\theta V_A}{d^3\sqrt[3]{\frac{m_A}{\rho_A}}}\right) = \frac{\emptyset V_B}{D^2\sqrt[3]{\frac{m_B}{\rho_B}}}$$

Rearranging the terms,

$$\boldsymbol{zn\left(\frac{\theta V_A}{d^3}\left(\sqrt[3]{\frac{\rho_A}{m_A}}\right)\right) = \frac{\emptyset V_B}{D^2}\left(\sqrt[3]{\frac{\rho_B}{m_B}}\right)} \qquad (8)$$

The value of T for planet B in terms of density and mass is,

$$T = \frac{H}{D} = \frac{\sqrt[3]{\frac{6m_B}{\pi\rho_B}}}{D} = \boldsymbol{\frac{1}{D}\sqrt[3]{\frac{6m_B}{\pi\rho_B}}}$$

To check the working of the expression we will substitute real values in the expression.

7.1 Example: Interplanetary Relationship between Earth and Mars

For Earth (Planet A) [3],
$\theta = 0.410\ rad$
$V_A = 465.1\ m/s$
$d = 149.6 * 10^9\ m$
$\rho_A = 5515\ Kgm^{-3}$
$m_A = 5.97 * 10^{24}\ Kg$

For Mars (Planet B) [4],
$\emptyset = 0.436\ rad$
$V_B = 241.7\ m/s$
$D = 227.9 * 10^9\ m$
$\rho_B = 3940\ Kgm^{-3}$
$m_B = 6.39 * 10^{23}\ Kg$

T for Mars,

$$T = \frac{1}{D}\sqrt[3]{\frac{6m_B}{\pi\rho_B}} = \frac{1}{227.9 * 10^9}\sqrt[3]{\frac{6 * 6.39 * 10^{23}}{\pi * 3940}}$$

$$= 3.00694 * 10^{-5}$$

Referring to Table 1 for values,

$$z = \frac{0.0061\, rad}{0.0093\, rad} = 0.65$$

Substituting the values in equation 8

LHS =

$$0.65\left(\frac{0.410 * 465.1}{(149.6 * 10^9)^3}\sqrt[3]{\frac{5515}{5.97 * 10^{24}}}\right) rad^1 m^{-3} s^{-1}$$

RHS =

$$\left(\frac{0.436 * 241.7}{(227.9 * 10^9)^2}\sqrt[3]{\frac{3940}{6.39 * 10^{23}}}\right) rad^1 m^{-2} s^{-1}$$

Simplifying the LHS and RHS by cancelling out common terms and units

LHS =

$$0.65\left(\frac{0.410 * 465.1}{(149.6)^3 * 10^9}\sqrt[3]{\frac{5515}{5.97 * 10^{24}}}\right) m^{-1}$$

RHS =

$$\left(\frac{0.436 * 241.7}{(227.9)^2}\sqrt[3]{\frac{3940}{6.39 * 10^{23}}}\right)$$

LHS = $3.52 * 10^{-21}\, m^{-1}$
RHS = $3.72 * 10^{-10}$

As T for Mars lies between $1.5 * 10^{-5}$ and $8.517 * 10^{-5}$ therefore using of rule number 1, RHS will be multiplied with $10^{-11}\, m^{-1}$.

$$\therefore RHS = 3.72 * 10^{-21}\, m^{-1}$$

And therefore RHS and LHS are nearly equal. Using this one example we have proved the working of equation 8.

8. INTRODUCTION TO NEW EXPRESSIONS FOR FINDING THE ANGULAR DIAMETER OF THE SUN FROM DIFFERENT PLANETS

For deriving the new expressions for angular diameter we need to make some changes in equation 7. We will rewrite z of equation 7 as the ratio of angular diameter of the sun as seen from planet B (δ_B) and the angular diameter of the sun as seen from planet A (δ_A), this will give us:

$$\frac{\delta_B}{\delta_A} * n\left(\frac{\theta V_A}{d^3\, h}\right) = \frac{\emptyset V_B}{D^2 H}$$

On making the ratio of the angular diameters as the subject we get,

$$\frac{\delta_B}{\delta_A} = \frac{\frac{\emptyset V_B}{D^2 H}}{n\left(\frac{\theta V_A}{d^3\, h}\right)}$$

We can rewrite this equation to form 2 independent equations to give,

$$\boldsymbol{\delta_A = n\left(\frac{\theta V_A}{d^3\, h}\right)\Lambda} \qquad (9)$$

$$\boldsymbol{\delta_B = \frac{\emptyset V_B}{D^2 H}\Lambda} \qquad (10)$$

Where Λ is a constant

Hence we have obtained 2 equations for the angular diameter of the sun. These 2 equations can only be used for finding the angular diameter of the sun from the planets.

Note: While using equation 10 we need to remember all the rules and range and multiply the RHS of equation 10 according to the value of T. This will ensure that the value and the units of constant Λ remain same for equation 9 and equation 10.

9. FINDING THE VALUE AND UNITS OF THE CONSTANT Λ

Like we have been doing, for finding the value of the constant Λ we will substitute real values in equations 9 and equations 10 for four planets. Then we will find the average which will give us a rough value of constant Λ.

9.1 Example: Substituting the Values for Earth in Equation 9

We will substitute all values in equation 9 and then make the constant the subject, which will give us the value of the constant.

For Earth [3],

$\theta = 0.410\ rad$
$V_A = 465.1\ m/s$
$d = 149.6 * 10^9\ m$
$h = 12742 * 10^3\ m$
$n = 1$
$\delta_A = 0.0093\ rad$

Substituting all these values with their units in equation 9,

$$0.0093\ rad = \Lambda\left(\frac{0.410 * 465.1}{(149.6 * 10^9)^3\ * 12742 * 10^3}\right) rad^1 m^{-3} s^{-1}$$

Making the constant Λ as the subject we get,

$$\Lambda = \frac{0.0093 * (149.6 * 10^9)^3\ * 12742 * 10^3}{0.410 * 465.1}\ m^3 s$$

$$\therefore \Lambda = 2.08058 * 10^{36}\ m^3 s$$

9.2 Example: Substituting the Values for Jupiter in Equation 9

We will substitute all values in equation 9 and then make the constant the subject, which will give us the value of the constant.

For Jupiter [6],
$\theta = 0.05462\ rad$
$V_A = 12600\ m/s$
$d = 776.82 * 10^9\ m$
$h = 139820 * 10^3\ m$
$\delta_A = 0.0018\ rad$
n = 79

Substituting all these values with their units in equation 9,

$$0.0018 = \Lambda\left(\frac{79 * 0.05462 * 12600}{(776.82 * 10^9)^3\ * 139820 * 10^3}\right) m^{-3} s^{-1}$$

Making the constant Λ as the subject we get,

$$\Lambda = \frac{0.0018 * (776.82 * 10^9)^3\ * 139820 * 10^3}{79 * 0.05462 * 12600}\ m^3 s$$

$$\therefore \Lambda = 2.16997 * 10^{36}\ m^3 s$$

9.3 Example: Substituting the Values for Neptune in Equation 10

We will substitute all values in equation 10 and then make the constant the subject, which will give us the value of the constant.

For Neptune [7],
$\emptyset = 0.4942\ rad$
$V_B = 2680\ m/s$
$D = 4.495 * 10^{12}\ m$
$H = 49244 * 10^3\ m$
$\delta_B = 0.00031\ rad$

T for Neptune,

$$T = \frac{H}{D} = \frac{49244 * 10^3}{4.495 * 10^{12}} = 1.09 * 10^{-5}$$

As T for Neptune doesn't lie between $1.5 * 10^{-5}$ and $8.517 * 10^{-5}$ therefore because of rule number 2, RHS will be multiplied with $10^{-10}\ m^{-1}$.

Substituting all values and their units in equation 10 we get,

$$0.00031\ rad = \Lambda \frac{0.4942 * 2680 * 10^{-10}}{(4.495 * 10^{12})^2 * 49244 * 10^3} rad^1 m^{-3} s^{-1}$$

Making the constant Λ as the subject,

$$\Lambda = \frac{0.00031 * (4.495 * 10^{12})^2 * 49244 * 10^3}{0.4942 * 2680 * 10^{-10}} m^3 s^1$$

$$\therefore \Lambda = 2.32882 * 10^{36}\ m^3 s^1$$

9.4 Example: Substituting the Values for Mercury in Equation 10

We will substitute all values in equation 10 and then make the constant the subject, which will give us the value of the constant.

For Mercury [9],
$\emptyset = 0.0005934\ rad$
$V_B = 3.026\ m/s$
$D = 50.726 * 10^9\ m$
$H = 4880 * 10^3\ m$
$\delta_B = 0.024\ rad$

T for Mercury,

$$T = \frac{H}{D} = \frac{4880 * 10^3}{50.726 * 10^9} = 9.62031 * 10^{-5}$$

As T for Mercury lies between $8.517 * 10^{-5}$ and $1 * 10^{-4}$ therefore because of rule number 3, RHS will be multiplied with $10^{-7}\ m^{-1}$.

Substituting all values and their units in equation 10 we get,

$$0.024\ rad$$

$$= \Lambda \frac{0.0005934 \ * 3.026 * 10^{-7}}{(50.726 * 10^9)^2 * 4880 * 10^3} rad^1 m^{-3} s^{-1}$$

Making the constant Λ as the subject,

$$\Lambda = \frac{(50.726 * 10^9)^2 * 0.024 * 4880 * 10^3}{0.0005934 \ * 3.026 * 10^{-7}} m^3 s^1$$

$$\therefore \Lambda = 1.67832 * 10^{36} \ m^3 s^1$$

The average value of constant Λ =

$$\frac{(2.08058 + 2.16997 + 2.32882 + 1.67832) * 10^{36}}{4}$$

$$\therefore \boldsymbol{\Lambda = 2.064 * 10^{36} \ m^3 s^1}$$

Hence we can use this constant and equation 9 and 10 to find the angular diameter of the sun.

10. APPLICATIONS OF THE NEW EXPRESSION OF ANGULAR DIAMETER

Like we did in the previous examples we will substitute real values in equation 9 and equation 10. This will help us see the application and verification of the equations.

10.1 Example: Calculating the Angular Diameter of the Sun from Earth Using Equation 9

In this example, we will substitute all the values except for the value of the angular diameter of the sun as seen from Earth in equation 9. Then we will find the value of angular diameter from the expression and compare it from the actual value.

For Earth [3],
$\theta = 0.410 \, rad$
$V_A = 465.1 \, m/s$
$d = 149.6 * 10^9 \ m$
$h = 12742 * 10^3 \ m$
$\Lambda = 2.064 * 10^{36} \ m^3 s^1$
$n = 1$

Substituting all these values and their units in equation 9 will gives us,

$$\delta_A = 1 \left(\frac{0.410 * 465.1 * 2.064 * 10^{36}}{(149.6 * 10^9)^3 \ * 12742 * 10^3} \right) rad$$

$$\delta_A = 0.00922587 \, rad$$

The actual value of the angular diameter of the sun from Earth is 0.0093 *rad*. Hence the equation 9 is working as the value we obtained from equation 9 is very close to the actual value.

10.2 Example: Calculating the Angular Diameter of the Sun from Mars Using Equation 10

In this case, we will substitute all values in equation 10 except for the value of the angular diameter of the sun as seen from Mars. The solution of this equation will give us the value of the angular diameter of the sun from Mars, and then we will compare the obtained value to from the actual value.

For Mars (Planet B) [4],
$\emptyset = 0.436 \, rad$
$V_B = 241.7 \, m/s$
$D = 227.9 * 10^9 \ m$
$H = 6779 * 10^3 \ m$
$\Lambda = 2.064 * 10^{36} \ m^3 s^1$

T for Mars,

$$T = \frac{H}{D} = \frac{6779 * 10^3}{227.9 * 10^9} = 2.97455 * 10^{-5}$$

As T for Mars lies between $1.5 * 10^{-5}$ and $8.517 * 10^{-5}$ therefore because of rule number 1, RHS will be multiplied with $10^{-11} \ m^{-1}$.

Substituting all these values and their units in equation 10 will gives us,

$$\delta_B = \frac{0.436 * 241.7 * 2.064 * 10^{36} * 10^{-11}}{(227.9 * 10^9 \,)^2 * 6779 * 10^3} \ rad$$

$$\delta_B = \ 0.00609 \, rad$$

The actual value of the angular diameter of the sun from Mars is 0.0061 *rad*. It is very close to the actual value. Hence the equation 10 is working.

We can see in both the examples that both equations are correct.

11. INTERPLANETARY RELATIONSHIP INDEPENDENT OF ANGULAR DIAMETER

In all the interplanetary relationship cases we saw that the expression was always depended on angular diameter of the sun as seen from that planet. But if we have a case where the angular diameter of the sun (from a planet) is unknown, for such cases we require an interplanetary relationship expression which is independent of angular diameter of the sun. For this, we will use the two equations of angular diameter. For this, we will substitute the value of angular diameter as the ratio of the diameter of the sun and the distance between the planet and the sun in equation 9 and equation 10.

Let the diameter of the sun be h_S

$$\delta \approx \frac{diameter\ of\ sun}{distance\ between\ the\ sun\ and\ the\ planet}$$

$$\therefore \delta_A = \frac{h_S}{d}\ rad$$

$$\therefore \delta_B = \frac{h_S}{D}\ rad$$

By substituting the value of δ_A in equation 9 will give us,

$$\frac{h_S}{d} = n\left(\frac{\theta V_A}{d^3\, h}\right)\Lambda$$

Cancelling out the common terms gives us,

$$h_S = n\left(\frac{\theta V_A}{d^2\, h}\right)\Lambda \qquad (11)$$

Similarly, substituting the value of δ_B in equation 10 will give us,

$$\frac{h_S}{D} = \frac{\emptyset V_B}{D^2 H}\Lambda$$

Cancelling out the common terms gives us,

$$h_S = \frac{\emptyset V_B}{DH}\Lambda \qquad (12)$$

Both equation 11 and 12 the units of h_S are $rad^1 m^1$. As the RHS of equation 11 and 12 are equal to the diameter of the sun, hence we can equate the LHS of equation 11 and 12, doing this will give us

$$n\left(\frac{\theta V_A}{d^2\, h}\right)\Lambda = \frac{\emptyset V_B}{DH}\Lambda$$

Cancelling out the common terms will give us,

$$\boldsymbol{n\left(\frac{\theta V_A}{d^2\, h}\right) = \frac{\emptyset V_B}{DH}} \qquad (13)$$

This is the expression for independent interplanetary relationship expression.

The meaning of each variable remains the same, also we will follow all the rules and range which we used for previous examples and the value of T remains the same.

12. APPLICATION AND EXAMPLE OF THE INDEPENDENT INTERPLANETARY RELATIONSHIP EXPRESSION

Example: We will compare Jupiter and Mars. We will take Jupiter on the LHS and Mars on the RHS.

For Jupiter (Planet A) [6],
$\theta = 0.05462\ rad$
$V_A = 12600\ m/s$
$d = 776.82 * 10^9\ m$
$h = 139820 * 10^3\ m$
$n = 79$

For Mars (Planet B) [4],
$\emptyset = 0.436\ rad$
$V_B = 241.7\ m/s$
$D = 227.9 * 10^9\ m$
$H = 6779 * 10^3\ m$

T for Mars,

$$T = \frac{H}{D} = \frac{6779 * 10^3}{227.9 * 10^9} = 2.97455 * 10^{-5}$$

Substituting all values in equation 13,

LHS =
$$79\left(\frac{0.05462 * 12600}{(776.82 * 10^9)^2\ * 139820 * 10^3}\right)\frac{rad}{m^2 s}$$

RHS =
$$\left(\frac{0.436 * 241.7}{227.9 * 10^9 * 6779 * 10^3}\right)\frac{rad}{m^1 s}$$

LHS = $6.44 * 10^{-28}\ rad^1 m^{-2} s^{-1}$
RHS = $6.82 * 10^{-17} rad^1 m^{-1} s^{-1}$

As T for Mars lies between $1.5 * 10^{-5}$ and $8.517 * 10^{-5}$ therefore because of rule number 1, RHS will be multiplied with $10^{-11}\ m^{-1}$.

$$\therefore RHS = 6.82 * 10^{-28} rad^1 m^{-2} s^{-1}$$

Hence the RHS and LHS are dimensionally and numerically nearly equal. Hence equation 13 is true.

13. THE RATIOS OF PHYSICAL CHARACTERISTICS OF THE PLANETS

In equation 11 and 12, we transfer all the constants on the LHS and all the variables on the RHS. Doing this will give us,

$$\frac{h_S}{\Lambda} = n\left(\frac{\theta V_A}{d^2\, h}\right) \tag{11}$$

$$\frac{h_S}{\Lambda} = \frac{\emptyset V_B}{DH} \tag{12}$$

For the value of the constant on the LHS,
$\Lambda = 2.064 * 10^{36}\ m^3 s^1$
$h_S = 1.3927 * 10^9\ \ rad\ m$

Therefore the LHS is,

$$\therefore \frac{h_S}{\Lambda} = \frac{1.3927 * 10^9}{2.064 * 10^{36}} = 6.74 * 10^{-28}\ rad^1 m^{-2} s^{-1}$$

Substituting the value of this constant in equation 11 and 12 will give us,

$$\mathbf{6.74 * 10^{-28}} = \boldsymbol{n}\left(\frac{\boldsymbol{\theta V_A}}{\boldsymbol{d^2\, h}}\right) \tag{14}$$

$$\mathbf{6.74 * 10^{-28}} = \frac{\boldsymbol{\emptyset V_B}}{\boldsymbol{DH}} \tag{15}$$

While using these equations we must substitute the units of the constant. Also, we need to multiply the RHS of equation 15 according to the rules and range of T which we formed for interplanetary relationship.

All planets follow equation 14 and 15. With the help of these 2 equations, we can find unknown values of any of the physical characteristics. They also show that the physical characteristics are not randomly taken up by the planets; they follow certain rules for taking up the physical characteristics. These rules are the equations 14 and 15. When one of the quantity increases the others have to compensate and decrease because they all are equal to a constant. They also show that all the physical characteristics of the planet are interrelated and change in any one of the quantities can affect the others. These equations explain why planets have unique physical characteristics.

Example of application: If we see the physical characteristics of Jupiter we will see that the axis tilt of Jupiter is very less compared to other planets in the outer solar system. This can be explained using equations 14 and 15. If we see all the other physical characteristics of Jupiter, then we can see that it has high equatorial rotational velocity and largest diameter. To compensate for all these large quantities one of the quantity had to decrease and that was axis tilt. Hence Jupiter has such less axis tilt.

A few planets don't exactly follow this ratio; they deviate from the actual value. This is because the values of their physical characteristics have errors in them and hence are not the exact values. When we form interplanetary relationships using these planets, then we don't get RHS and LHS equal to each other. The planets which give a high value of the difference of RHS and LHS in the interplanetary relationship, those planets don't give accurate value using equation 14 and 15. This is because of the high error in the values of physical characteristics of these planets.

14. CONCLUSIONS

In all examples, we saw that the interplanetary relationship and the expression for angular diameter hold. At this moment I don't have any explanation for the working of the interplanetary relationship and the expression for angular diameter. I think the expressions which we discussed in the paper are a fundamental property of our solar system. The interplanetary relationship shows that the arrangement of planets and even the number of moons a planet can have are directed by rules, which in this case was the interplanetary relationship expression. The interplanetary expression can help us find accurate physical characteristics of the planets; it can also help us in understanding the formation of planets. Relations which bind independent quantities show the beauty of mathematics and our cosmos. This paper also shows the natural order which the planets follow.

15. SIGNIFICANCE OF THE PAPER

The highlights of the paper are the interplanetary relationship expression and the two new expressions for the angular diameter of the sun. Using the interplanetary relationship we can accurately find the values of distances between planets and the sun, diameter of planets, axis tilts of planets, equatorial rotational velocities and the number moons a planet can have. The expression explains the position of planets in our solar system. Using the equation we can also predict the formation of the planets in our solar system. Using the equations of angular diameter, we can accurately find the angular diameter of the sun from any planet. The new equation for angular diameter shows more quantities on which the angular diameter of the sun is dependent. These two highlights mentioned above our revolutionary ways of looking at our solar system.

16. SUMMARY

In this paper, we have concluded that the expression which we derived in the paper with the title "Expression for the relative change of height when the distance between the viewer and the body changes; and its cosmological application" is wrong and incomplete.

In this paper, we also saw expressions which showed us the relation between diameters, axis tilts, equatorial rotational velocities and distances of two planets, which depended upon the ratio of angular diameter of the sun as seen from the planets. In this interplanetary relationship expression, we saw two cases; in the first case, we discussed about an expression using which we compared the size of the sun from all the planets with the size of the visible from Earth, which we called the special case. In the second case, we obtained an expression with which we could form relations between any two planets of our solar system.

The special case expression is

$$z\left(\frac{\theta V_A}{d^3\, h}\right) = \frac{\emptyset V_B}{D^2 H}$$

The general case expression is

$$zn\left(\frac{\theta V_A}{d^3\, h}\right) = \frac{\emptyset V_B}{D^2 H}$$

We also saw that the RHS and LHS of the expression were not dimensionally and numerically balanced, so to balance out the RHS and LHS we introduced some rules and a range which are as follows:

1. If the value of T for a planet is greater than $1.5 * 10^{-5}$ and less than $8.517 * 10^{-5}$ then the RHS will be multiplied by $10^{-11}\, m^{-1}$

2. If the value of T for a planet does not lie between $1.5 * 10^{-5}$ and $8.517 * 10^{-5}$ then RHS will be multiplied by $10^{-10}\, m^{-1}$

3. If the value of T for a is greater than $8.517 * 10^{-5}$ and less than $1 * 10^{-4}$ then RHS will be multiplied by $10^{-7}\, m^{-1}$

4. If the value of T for a planet is equal to $8.517 * 10^{-5}$ (for Earth) then the RHS is multiplied by $10^{-11}\, m^{-1}$

Here T is the ratio of the diameter of planet B and the distance between the sun and planet B. Using the general case equation of the interplanetary relationship we obtained two new expressions for the angular diameter of the sun. Which are:

$$\delta_A = n\left(\frac{\theta V_A}{d^3\, h}\right)\Lambda$$

$$\delta_B = \frac{\emptyset V_B}{D^2 H}\Lambda$$

Here, Λ is a constant and the value of the constant is

$$\Lambda = 2.064 * 10^{36}\, m^3 s^1$$

For the second equation of angular diameter, we will multiply the RHS with $10^{-11}\, m^{-1}$ or $10^{-10}\, m^{-1}$ or $10^{-7}\, m^{-1}$ according to the rules and range we formed for interplanetary relationship.

Using the 2 equations of angular diameter we also derived an interplanetary relationship expression which is independent of angular diameter. The equation is

$$n\left(\frac{\theta V_A}{d^2\, h}\right) = \frac{\emptyset V_B}{DH}$$

All the quantities are the same as the interplanetary relationship expression. We will follow all the rules and range which we formed for the general and special case of interplanetary relationship.

Using all these expressions we can revolutionize our understanding of our solar system.

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Author has declared that no competing interests exist.

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