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To Correlate Galactic Dark and Visible Masses and to Fit Flat Rotation Speeds Via MOND Approach and Cosmic Angular Acceleration

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Authors' contributions

This work was carried out in collaboration between both authors. Author UVSS designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author SL managed the analyses of the study and managed the literature searches. Both authors read and approved the final manuscript.

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ABSTRACT

Considering our recently proposed light speed expanding and rotating primordial black hole universe and by following Modified Newtonian Dynamics (MOND), an attempt is made to estimate the galactic dark mass and galactic flat rotation curves. Basic idea is that, galactic dark mass is a representation of weakly interacting massive foam and its magnitude is proportional to $(\text{galactic visible mass})^{3/2}$. Considering current cosmic maximum angular acceleration, MOND's approach implicitly seems to support the cosmological estimation of 95% invisible matter and 5% visible matter. With reference to Metric Skew Tensor Gravity (MSTG) and MOND approaches, in fitting the galactic flat rotation curves, for 101 galaxies, average error is -2.1% and 6.8% respectively. Estimated total mass of Milky Way is 2.28 Trillion solar masses and is matching with the upper mass limit of most recent studies.

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Keywords: Cosmic angular velocity; Galactic visible mass; Galactic dark mass; Galactic total mass; Galactic effective radius; Galactic visible radius; Galactic core radius.

1. INTRODUCTION

In our recent publication [1], an attempt has been made to develop a practical model of cosmology. Main features of our integrated model are: eternal role of Planck scale, light speed expansion and rotation of a primordial cosmic black hole, slow thermal cooling, internal acceleration and anisotropy. At any stage of cosmic expansion, there exists a tight correlation between cosmic angular velocity and cosmic temperature. At $H_0 \cong 70\ \text{km/Mpc/sec},$ present angular velocity seems to be 140.56 times smaller than the Hubble parameter. In this review paper, an attempt is made:

1) To infer galactic dark mass [2,3] as a representation of weak interaction invisible massive foam.
2) To estimate Galactic dark mass factor with 'proton mass' and weak interaction.
3) To analyze MOND relation (Modified Newtonian Dynamics) [4], in terms of orbiting velocity rather than escape velocity [1].
4) To retain the idea of implementing current cosmic maximum angular acceleration.

2. MOND APPROACH OF ESTIMATING GALACTIC FLAT ROTATION CURVES

Originally, MOND was introduced for fitting the observed galactic flat orbiting speeds of stars without the aid of dark matter. As per the MOND, gravity takes on a specific non-Newtonian form at accelerations below a definite universal value. MOND formula is very simple and constitutes only one fixed parameter called the 'critical acceleration'. MOND is working in well in estimating the spiral galaxy rotation curves from the observed distribution of visible matter. Point to be noted is that MOND makes no predictions or explanation with respect to cosmology and galactic structures. Another problem is that, the term critical acceleration, $1.2\times10^{-10}\ \text{m.sec}^{-2}$ is quantitatively less than $\left(cH_0\right).$

3. BASIC ASSUMPTIONS

We propose the following three assumptions.

1) Galactic dark mass increases with increasing galactic visible mass.
2) Galactic dark mass increases with increasing galactic radius.
3) Cosmic angular velocity plays a vital role in galactic rotation curves.

4. UNDERSTANDING AND ESTIMATING DARK MATTER

As per modern cosmological observations, most of the cosmologists infer dark matter as a characteristic and inherent feature of any galaxy. Dark matter seems to have a major role in understanding 6 different issues pertaining to many of the galaxies. They are:

a) Galactic formation and evolution.
b) Galactic rotational curves.
c) Gravitational lensing.
d) Galactic collisions.
e) Motion of galaxies within galaxy clusters.
f) Cosmic microwave fluctuations.

Most unfortunate thing is that, so far, no ground based experiment or no cosmological observation could establish any direct evidence for the existence of dark matter and opened a new window for MOND like interesting concepts. In this context, some of the cosmologists are trying to understand the presumed 6 major applications of dark matter with galactic "visible mass" only. But, effectiveness of this attempt seems to be poor and is in its budding stage. Here we would like appeal that, the subject under consideration is falling under a 'debate' and needs further study at utmost fundamental level with respect to the strange nature of dark matter. Ongoing and future experiments and observations may help in resolving the issue. One can find interesting technical discussion in the context of galactic rotation curves with a 'variable' mass to light ratio [5].

For the time being, keeping the 6 major applications of dark matter in view, we make an attempt to estimate the generally believed dark mass of a galaxy with its corresponding visible mass. In this context, we consider MOND relation as an ideal tool for understanding and verification via cosmic angular acceleration. One most interesting as well as speculative point is that, even though MOND approach is 'the best' in fitting galactic rotation curves, its back ground physics is unclear with respect to galactic structures and cosmic acceleration parameter

(cH_0). It can be confirmed with the conclusion section of recent reference [6].

The most interesting point to be noted is that, in all of the multiple applications, role of dark matter seems to be 'a key agent of gravity' having large mass. Another interesting point to be noted is that, most of the scientists believe that, dark matter is somehow connected with 'weak' interaction. Considering these points in view, we would like to appeal that, galactic dark matter can be considered as a representation of weakly interacting massive foam responsible for binding its surrounding visible matter via gravity. Quantitatively it can be estimated with the following semi empirical relation. By trial-error we have developed this relation. It needs in depth discussion at basic level. For the time being, we appeal the readers to consider it as a quantitative fit.

$$\frac{G_w M_{dark}}{G M_{vis}} \cong \sqrt{\frac{M_{vis}}{m_p}} \tag{1}$$

where,

M_{dark} = Dark mass of galaxy. We are working on its scope of applicability for elementary particles, massive stars and other compact celestial objects.

M_{vis} = Visible or observable mass of galaxy or star or any elementary particle.

G = Newtonian gravitational constant.

$\cong 6.67408\times10^{-11}\ \text{m}^3.\text{kg}^{-1}.\text{sec}^{-2}$

G_w = Gravitational constant connected with weak interaction.

$\cong 2.909745\times10^{22}\ \text{m}^3.\text{kg}^{-1}.\text{sec}^{-2}$

m_p = Proton rest mass

For more details on G_w, readers are encouraged to see our recently published papers on Weak gravitational constant [7,8,9].

Based on relation (1), let

X_{dark} = Ratio of dark mass to visible mass of galaxy

= Dark mass factor.

$$\begin{aligned} X_{dark} &\cong \frac{M_{dark}}{M_{vis}} \cong \left(\frac{G}{G_w}\right)\sqrt{\frac{M_{vis}}{m_p}} \\ &\cong \sqrt{\frac{M_{vis}}{(G_w/G)^2 m_p}} \cong \sqrt{\frac{\text{M}_{vis}}{3.179\times10^{38}\ \text{kg}}} \end{aligned} \tag{2}$$

$$M_{dark} \cong X_{dark} M_{vis} \cong \frac{(M_{vis})^{3/2}}{\sqrt{3.179\times10^{38}\ \text{kg}}} \tag{3}$$

where, $(G_w/G)^2 m_p \cong 3.179\times10^{38}\ \text{kg} \cong M_{\text{Ref}}$ can be called as 'Reference mass unit'.

Based on relation (3),

$$M_{dark} \propto (M_{vis})^{\frac{3}{2}} \tag{4}$$

In this paper, we try to understand the effectiveness of relation (3) with respect to galactic visible mass as a whole.

5. TO ESTIMATE GALACTIC DARK MASS AND TOTAL MASS

Galactic total mass can be estimated as follows.

With the following empirical relation, we try to estimate the dark mass of galaxy. It needs further study with respect to ultra faint dwarf galaxies (believed to have more dark matter) and their actual galactic rotation curves.

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$$\begin{aligned} M_{dark} &\cong X_{dark} * M_{vis} \\ &\cong \frac{(M_{vis})^{3/2}}{\sqrt{3.179\times10^{38}\ \text{kg}}} \end{aligned} \tag{5}$$

where, M_{vis} = Estimated visible mass of galaxy.

With the following relation, total mass of galaxy can be estimated.

$$\begin{aligned} M_{total} &\cong M_{dark} + M_{vis} \\ &\cong X_{dark} M_{vis} + M_{vis} \cong (X_{dark}+1) M_{vis} \end{aligned} \tag{6}$$

$$\%M_{dark} \cong \frac{X_{dark}}{(X_{dark}+1)} \times 100 \tag{7}$$

6. TO DEVELOP A MOND LIKE RELATION FOR GALACTIC FLAT ORBITING SPEED WITH COSMIC ANGULAR VELOCITY AND GALACTIC TOTAL MASS

Observed galactic flat rotation curves can be understood in the following way.

At present, based on the observed flat rotation speed and existence of dark matter, for any galaxy, let,

$$V_{orb} \cong \sqrt{\frac{GM_1}{r_1}} \cong \sqrt{\frac{GM_2}{r_2}} \cong \sqrt{\frac{GM_3}{r_3}} \tag{8}$$

where,

$V_{orb} \cong$ Observed flat orbiting velocity of galactic star.

$r_1, r_2, r_3 \cong$ Increasing galactic distances from galactic center.

$M_1, M_2, M_3 \cong$ Increasing galactic masses at r_1, r_2, r_3.

$$\left(\frac{M_1}{r_1} \cong \frac{M_2}{r_2} \cong \frac{M_3}{r_3} \cong \text{Constant}\right)$$

Point to be noted is that, star's orbiting velocity may change with changing galactic dark mass distribution and it needs further study and observational data for a number of galaxies. In that case, relation (8) needs a minor revision.

Let,

$$V_{orb} \cong \sqrt{\frac{GM_{total}}{r_{effe}}} \tag{9}$$

where,

$V_{orb} \cong$ Observed flat orbiting velocity of galactic star.

$M_{total} \cong$ Galactic total mass.

$r_{effe} \cong$ Galactic effective radius.

Writing $r_{effe} \cong \frac{GM_{total}}{V_{orb}^2}$ and eliminating r_{effe},

$$\frac{GM_{total}}{r_{effe}^2} \cong GM_{total}\left(\frac{GM_{total}}{V_{orb}^2}\right)^{-2} \cong \frac{V_{orb}^4}{GM_{total}} \tag{10}$$

Now, based on MOND approach, assume that,

$$\frac{V_{Orb}^4}{GM_{total}} \cong c\omega_0 \tag{11}$$

where,

$\omega_0 \cong$ Current cosmic angular velocity.

$\cong 1.61394 \times 10^{-20}$ rad/sec

$c\omega_0 \cong$ Current possible maximum cosmic angular acceleration.

Thus,

$$V_{orb} \cong \sqrt[4]{GM_{total}c\omega_0} \tag{12}$$

7. TO FIT GALACTIC FLAT ROTATION SPEEDS

Based on relations (5) and (12), observed galactic rotation speeds can be fitted with the following relation.

$$V_{orb} \cong \sqrt[4]{G\left[\left(X_{dark}+1\right)M_{vis}\right]c\omega_0} \tag{13}$$

Corresponding MOND formula is,

$$V_{orb} \cong \sqrt[4]{G\ \left(24.77M_{vis}\right)c\omega_0}$$
$$\text{where, } \left(\frac{1.2\times10^{-10}\ \text{m.sec}^{-2}}{c\omega_0}\right) \cong 24.77 \tag{14}$$

Proceeding further, galactic angular velocity can be defined with the following relation.

$$\omega_{gal} \cong \frac{V_{Orb}^3}{GM_{total}} \tag{15}$$

This is for observational verification. Now, it is possible to say that,

$$V_{orb}\omega_{gal} \cong c\omega_0 \tag{16}$$

See Figs. 1-3 and Table 1. In Figs. 1- 3 and Table 1, considering Metric Skew Tensor Gravity (MSTG) masses as a common reference [10,11], blue curve indicates (MSTG) rotation speeds, black curve indicates rotation speed estimated from MOND formula and red curve indicates the rotation speeds estimated with our proposed relation (13). In estimating flat rotation speeds, in case of MSTG, for 25 Dwarf galaxies, 17 low surface brightness (LSB) galaxies and 58 high surface brightness HSB galaxies, obtained errors are -3.8%, -6.4% and -0.1% respectively. In case of MOND, obtained errors are 17.9%, 10.2% and 0.9% respectively.

8. DISCUSSION

Based on this procedure, we would like to appeal that,

1) As 'spin' is a basic property of quantum mechanics, from the subject point of quantum gravity, universe must have

‘rotation’. But progress in quantum cosmology is poor [12].

2) Very recent and advanced studies of Lior Shamir suggest [13] that, the distribution of galaxy spin directions in SDSS and Pan-STARRS shows patterns in the asymmetry between galaxies with opposite spin directions and can be considered as an evidence for large-scale anisotropy and an indication for a rotating universe.
3) Even though MOND approach was aimed for understanding galactic rotation curves without dark matter, with reference to the proposed current cosmic angular velocity and relation (10), it is possible to fit the rotation curves and thereby galactic dark masses can be inferred.
4) On comparison, percentage of dark mass in MOND model seems to be constant at (23.77/24.77)x100 = 95.96% whereas in our approach, dark matter percentage increases with increasing (visible) mass and radius of galaxy. It is very interesting to note that, MOND’s approach implicitly seems to support the cosmological estimation of 95% invisible matter and 5% visible matter. It needs further study.

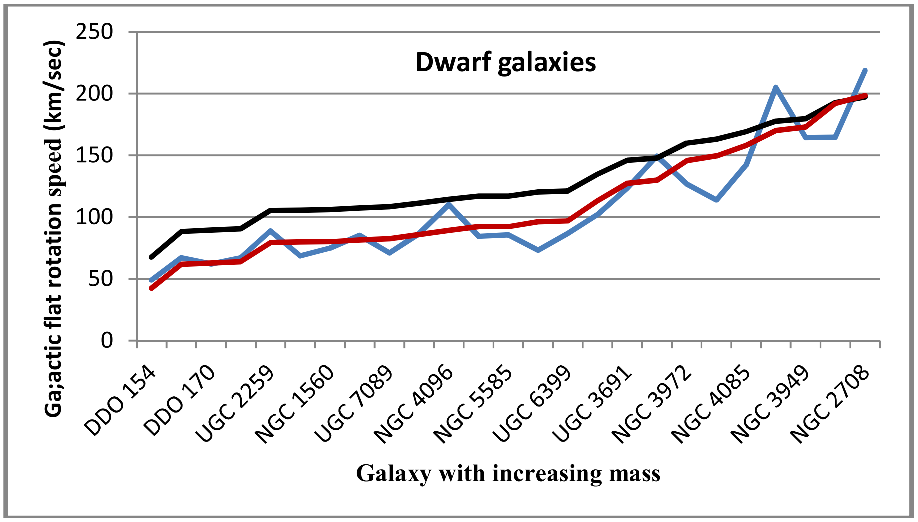

Fig. 1. Galactic dark masses and flat rotation speeds of Dwarf galaxies

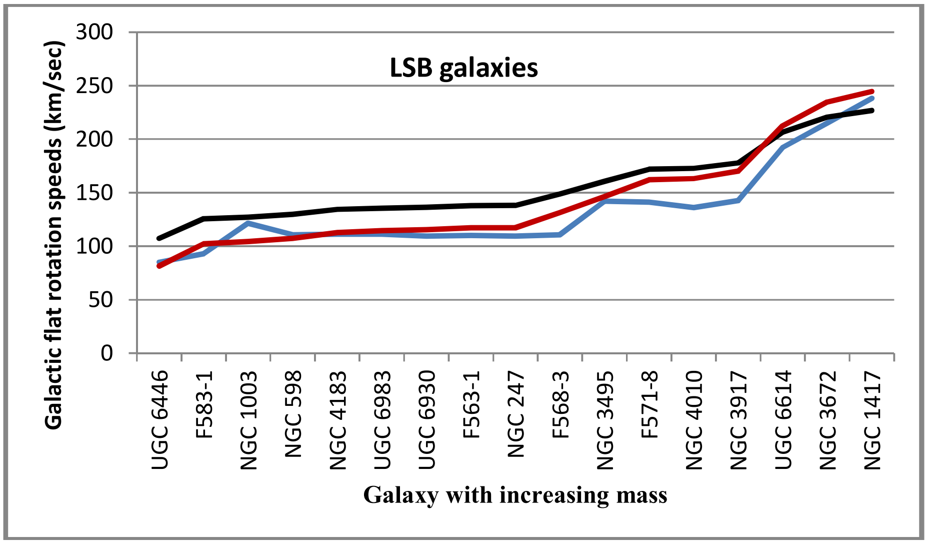

Fig. 2. Galactic dark masses and flat rotation speeds of LSB galaxies

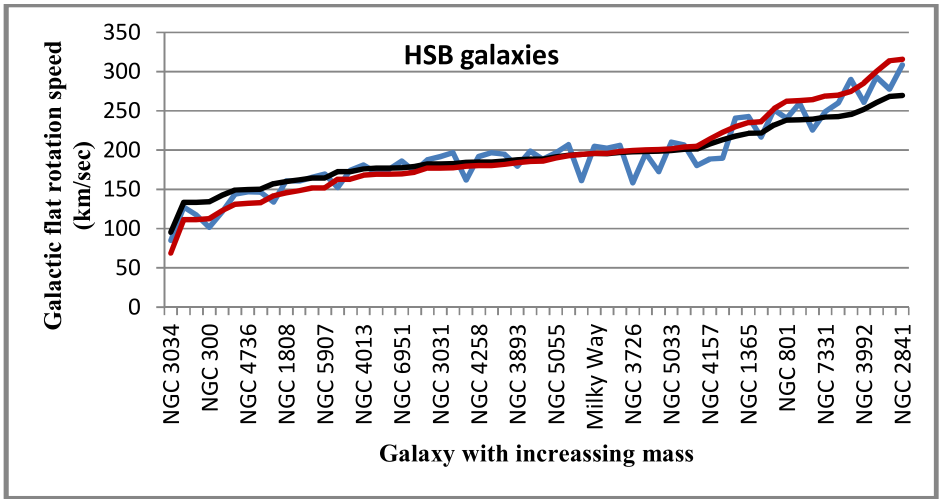

Fig. 3. Galactic dark masses and flat rotation speeds of HSB galaxies

5) From Table 1, it is clear that, in MOND model, assumed dark matter percentage is on higher side for low massive galaxies causing high errors. It needs further study.
6) Staring from the lowest massive galaxy, (DDO 154) to the highest massive galaxy (NGC 2841), dark mass seems to increase from 3.0 to 46 times respectively and needs further study. Applying this idea to Sun like stars, dark mass ratio is close to 0.0001.
7) As per the recent studies [14], Virial mass of Milky Way is $1.28^{+0.97}_{-0.48}\times10^{12}M_{Sun}$ and its corresponding upper limit is $2.25\times10^{12}M_{Sun}$. Based on relations (5) and (13), for Milky Way [8], estimated flat rotation speed is 195.8 km/sec and its corresponding total mass is $25\times\left[9.12\times10^{10}M_{Sun}\right]\cong2.28\times10^{12}M_{Sun}$. This is a good fit and strong support for our proposal. Based on relation (15), estimated angular velocity of Milky Way is 2.47×10^{-17} m.sec^{-2}. It is for observational testing.
8) Based on relations (5), (9) and (12), effective radius of galaxy can be expressed as,

$$r_{effe}\cong\frac{GM_{total}}{\sqrt{G\left[\left(X_{dark}+1\right)M_{vis}\right]c\omega_0}}\cong\sqrt{\frac{GM_{total}}{c\omega_0}} \qquad (17)$$

9) Based on relation (8), as a special case, radius of galaxy corresponding to its visible mass and flat rotation speed, can be called as galactic 'visible radius' and can be expressed as,

$$r_{vis}\cong\frac{GM_{vis}}{\sqrt{G\left[\left(X_{dark}+1\right)M_{vis}\right]c\omega_0}}\cong\sqrt{\frac{GM_{vis}}{\left(X_{dark}+1\right)c\omega_0}} \qquad (18)$$

10) Based on relations (8), (17) and (18), if dark matter distribution is 'as expected', galaxy should follow flat rotation speeds in between r_{effe} and r_{vis}. A least, close to the geometric mean of $\left(r_{vis}\right)$ and $\left(r_{effe}\right)$ rotation speed should be flat. It can be expressed as,

$$r_{geom}\cong\sqrt{\left(r_{vis}\right)\left(r_{effe}\right)}\cong\sqrt{\frac{GM_{vis}}{c\omega_0}} \qquad (19)$$

11) Effective, geometric and visible radii can be expressed with a common relation of the form,

$$r_{vis}\cong\left(X_{dark}+1\right)^p\sqrt{\frac{GM_{vis}}{c\omega_0}} \qquad (20)$$
$$\text{where, } p=\left(+\frac{1}{2},0,-\frac{1}{2}\right)$$

12) For Milky Way, its corresponding 'visible' and 'effective' radii are 10.3 kpc and 256.9 kpc. Corresponding geometric radius is 51.4 kpc. As per the observational data [15], for Milky Way, starting from a radius of 60 kpc, rotation speed seems to decrease gradually [16,17].

Table 1. Estimation of galactic dark masses and rotation speeds

Galaxy Name	Galaxy visible mass (kg)	Dark mass factor X_{dark}	Dark mass%	Rotation speed from MSTG estimations (km/sec)	Rotation speed from MOND estimations (km/sec) Relation(14)	Estimated rotation speed (km/sec) Relation(13)	%Error w.r.t MSTG	%Error w.r.t MOND
Dwarf (LSB & HSB) Galaxies								
DDO 154	2.6E+39	2.9	74.1	48.9	67.5	42.4	13.2	37.2
F583-4	7.6E+39	4.9	83.0	67.2	88.3	61.7	8.2	30.2
DDO 170	8E+39	5.0	83.4	61.9	89.4	62.8	-1.5	29.8
DDO 168	8.4E+39	5.1	83.7	67.1	90.5	63.9	4.8	29.4
UGC 2259	1.54E+40	7.0	87.4	88.8	105.4	79.3	10.7	24.7
NGC 3109	1.56E+40	7.0	87.5	68.6	105.7	79.7	-16.2	24.6
NGC 1560	1.58E+40	7.1	87.6	74.9	106.0	80.1	-6.9	24.5
UGC 6446	1.66E+40	7.2	87.9	85.1	107.3	81.5	4.2	24.1
UGC 7089	1.72E+40	7.4	88.0	71.1	108.3	82.6	-16.1	23.8
UGC 6923	1.92E+40	7.8	88.6	86.5	111.3	85.9	0.7	22.8
NGC 4096	2.14E+40	8.2	89.1	110.1	114.4	89.3	18.9	21.9
NGC 55	2.34E+40	8.6	89.6	84.4	117.0	92.3	-9.3	21.1
NGC 5585	2.34E+40	8.6	89.6	85.7	117.0	92.3	-7.7	21.1
UGC 6818	2.62E+40	9.1	90.1	73.1	120.3	96.1	-31.5	20.1
UGC 6399	2.68E+40	9.2	90.2	86.7	121.0	96.9	-11.8	19.9
UGC 6917	4.12E+40	11.4	91.9	102.1	134.7	113.3	-11.0	15.9
UGC 3691	5.66E+40	13.4	93.0	123.5	145.9	127.3	-3.1	12.8
NGC 4062	5.96E+40	13.7	93.2	149.4	147.8	129.7	13.2	12.2
NGC 3972	8.18E+40	16.1	94.1	126.8	159.9	145.7	-14.9	8.9
NGC 4389	8.8E+40	16.7	94.3	113.9	162.9	149.7	-31.4	8.1
NGC 4085	1.02E+41	17.9	94.7	142	169.1	158.1	-11.4	6.5
NGC 4569	1.25E+41	19.8	95.2	205	177.7	170.1	17.0	4.3
NGC 3949	1.3E+41	20.3	95.3	164.5	179.6	172.9	-5.1	3.8
NGC 3877	1.73E+41	23.4	95.9	164.8	192.9	192.1	-16.5	0.4
NGC 2708	1.89E+41	24.4	96.1	218.7	197.1	198.3	9.3	-0.6
LSB Galaxies								
UGC 6446	1.66E+40	7.2	87.9	85.1	107.3	81.5	4.2	24.1

Galaxy Name	Galaxy visible mass (kg)	Dark mass factor X_{dark}	Dark mass%	Rotation speed from MSTG estimations (km/sec)	Rotation speed from MOND estimations (km/sec) Relation(14)	Estimated rotation speed (km/sec) Relation(13)	%Error w.r.t MSTG	%Error w.r.t MOND
F583-1	3.12E+40	9.9	90.8	93.2	125.7	102.4	-9.9	18.5
NGC 1003	3.28E+40	10.2	91.0	121.5	127.3	104.3	14.2	18.1
NGC 598	3.56E+40	10.6	91.4	110.9	129.9	107.4	3.1	17.3
NGC 4183	4.08E+40	11.3	91.9	111.3	134.4	112.9	-1.5	16.0
UGC 6983	4.24E+40	11.6	92.0	111.5	135.7	114.5	-2.7	15.6
UGC 6930	4.34E+40	11.7	92.1	109.5	136.5	115.5	-5.5	15.4
F563-1	4.52E+40	11.9	92.3	110.4	137.9	117.2	-6.2	15.0
NGC 247	4.54E+40	12.0	92.3	109.4	138.1	117.4	-7.3	15.0
F568-3	6.16E+40	13.9	93.3	110.9	149.0	131.3	-18.4	11.9
NGC 3495	8.32E+40	16.2	94.2	142.1	160.6	146.6	-3.2	8.7
F571-8	1.09E+41	18.6	94.9	141.2	171.9	162.0	-14.8	5.7
NGC 4010	1.11E+41	18.7	94.9	136.2	172.7	163.1	-19.8	5.5
NGC 3917	1.25E+41	19.8	95.2	142.8	177.8	170.3	-19.3	4.2
UGC 6614	2.27E+41	26.8	96.4	192.3	206.5	212.4	-10.5	-2.9
NGC 3672	2.97E+41	30.6	96.8	215.2	220.8	234.7	-9.1	-6.3
NGC 1417	3.32E+41	32.3	97.0	238.2	227.0	244.5	-2.7	-7.7
HSB Galaxies								
NGC 3034	1.04E+40	5.7	85.1	85	95.5	68.9	18.9	27.8
NGC 4448	3.96E+40	11.2	91.8	127.8	133.4	111.7	12.6	16.3
NGC 6503	3.96E+40	11.2	91.8	117.4	133.4	111.7	4.9	16.3
NGC 300	4.06E+40	11.3	91.9	101.7	134.2	112.7	-10.8	16.0
NGC 3769	5.18E+40	12.8	92.7	121.7	142.7	123.2	-1.2	13.6
NGC 4303	6.16E+40	13.9	93.3	143.8	149.0	131.3	8.7	11.9
NGC 4736	6.3E+40	14.1	93.4	146.8	149.8	132.4	9.8	11.7
NGC 660	6.4E+40	14.2	93.4	146.6	150.4	133.1	9.2	11.5
NGC 2403	7.6E+40	15.5	93.9	133.7	157.0	141.8	-6.1	9.7
NGC 1808	8.2E+40	16.1	94.1	160.6	160.0	145.8	9.2	8.9
NGC 4138	8.62E+40	16.5	94.3	160.7	162.1	148.5	7.6	8.3
NGC 4945	9.16E+40	17.0	94.4	165.1	164.5	151.9	8.0	7.7
NGC 5907	9.18E+40	17.0	94.4	169.3	164.6	152.0	10.2	7.7

Galaxy Name	**Galaxy visible mass (kg)**	**Dark mass factor** X_{dark}	**Dark mass%**	**Rotation speed from MSTG estimations (km/sec)**	**Rotation speed from MOND estimations (km/sec) Relation(14)**	**Estimated rotation speed (km/sec) Relation(13)**	**%Error w.r.t MSTG**	**%Error w.r.t MOND**
NGC 3198	1.11E+41	18.7	94.9	152.1	172.6	163.0	-7.2	5.6
NGC 4527	1.11E+41	18.7	94.9	174.3	172.6	163.0	6.5	5.6
NGC 4013	1.2E+41	19.5	95.1	181.1	176.1	167.9	7.3	4.7
NGC 4631	1.23E+41	19.7	95.2	171.4	177.1	169.3	1.2	4.4
NGC 5236	1.23E+41	19.7	95.2	175.5	177.2	169.4	3.5	4.4
NGC 6951	1.24E+41	19.8	95.2	185.8	177.6	170.0	8.5	4.3
UGC 6973	1.28E+41	20.1	95.3	172.5	179.0	171.9	0.3	3.9
NGC 253	1.39E+41	20.9	95.4	188	182.5	177.0	5.8	3.0
NGC 3031	1.39E+41	20.9	95.4	191.8	182.6	177.1	7.6	3.0
NGC 3379	1.4E+41	21.0	95.5	196.7	182.9	177.5	9.8	2.9
NGC 4051	1.44E+41	21.3	95.5	161.7	184.3	179.5	-11.0	2.6
NGC 4258	1.46E+41	21.4	95.5	191.9	184.8	180.3	6.1	2.4
NGC 5194	1.46E+41	21.4	95.5	196.6	184.8	180.3	8.3	2.4
NGC 891	1.49E+41	21.7	95.6	194.9	185.9	181.9	6.7	2.2
NGC 3893	1.54E+41	22.0	95.7	179.3	187.3	184.0	-2.6	1.8
NGC 3521	1.58E+41	22.3	95.7	198.7	188.5	185.6	6.6	1.5
IC 342	1.59E+41	22.4	95.7	188.3	188.9	186.2	1.1	1.4
NGC 5055	1.68E+41	23.0	95.8	196.9	191.4	189.8	3.6	0.8
NGC 3079	1.75E+41	23.5	95.9	207.1	193.3	192.7	7.0	0.3
NGC 6946	1.79E+41	23.8	96.0	161.2	194.5	194.5	-20.7	0.0
Milky Way	**1.82E+41**	**24.0**	**96.0**	**204.8**	**195.4**	**195.8**	**4.4**	**-0.2**
NGC 3628	1.83E+41	24.0	96.0	202.3	195.5	195.9	3.2	-0.2
NGC 1068	1.88E+41	24.4	96.1	205.9	197.0	198.2	3.7	-0.6
NGC 3726	1.92E+41	24.6	96.1	158.4	198.0	199.6	-26.0	-0.8
NGC 2903	1.93E+41	24.7	96.1	195.9	198.3	200.1	-2.1	-0.9
NGC 4088	1.95E+41	24.8	96.1	172.4	198.7	200.7	-16.4	-1.0
NGC 5033	1.98E+41	25.0	96.2	210.2	199.5	201.9	4.0	-1.2
NGC 5457	2.04E+41	25.4	96.2	206.5	201.0	204.1	1.1	-1.6
NGC 4100	2.06E+41	25.5	96.2	180.2	201.5	204.9	-13.7	-1.7
NGC 4157	2.33E+41	27.1	96.4	188.5	207.7	214.4	-13.7	-3.2

Galaxy Name	Galaxy visible mass (kg)	Dark mass factor X_{dark}	Dark mass%	Rotation speed from MSTG estimations (km/sec)	Rotation speed from MOND estimations (km/sec) Relation(14)	Estimated rotation speed (km/sec) Relation(13)	%Error w.r.t MSTG	%Error w.r.t MOND
NGC 4217	2.58E+41	28.5	96.6	189.7	213.2	222.8	-17.5	-4.5
NGC 2590	2.81E+41	29.8	96.7	241	217.7	229.9	4.6	-5.6
NGC 1365	2.99E+41	30.7	96.8	242.6	221.2	235.3	3.0	-6.4
NGC 2998	3.03E+41	30.9	96.9	216.7	221.8	236.3	-9.0	-6.5
NGC 4565	3.62E+41	33.8	97.1	251.2	232.0	252.6	-0.5	-8.9
NGC 801	4.01E+41	35.6	97.3	240.3	238.0	262.4	-9.2	-10.2
NGC 224	4.04E+41	35.7	97.3	259.6	238.4	263.0	-1.3	-10.3
NGC 3953	4.09E+41	35.9	97.3	225.5	239.2	264.3	-17.2	-10.5
NGC 7331	4.29E+41	36.8	97.4	248.9	242.1	269.0	-8.1	-11.1
NGC 4321	4.33E+41	37.0	97.4	260.2	242.7	270.0	-3.8	-11.3
NGC 1097	4.54E+41	37.8	97.4	290.1	245.4	274.6	5.3	-11.9
NGC 3992	5.03E+41	39.8	97.6	260.9	251.9	285.4	-9.4	-13.3
NGC 5533	5.76E+41	42.6	97.7	293.2	260.6	300.1	-2.4	-15.2
NGC 6674	6.5E+41	45.2	97.8	277.7	268.5	313.8	-13.0	-16.9
NGC 2841	6.61E+41	45.6	97.9	308.3	269.6	315.8	-2.4	-17.1

Table 2. Estimated galactic visible and effective radii

Galaxy Name	Galaxy visible mass (kg)	Dark mass factor X_{dark}	Dark mass%	Estimated rotation speed (km/sec) Relation (13)	Galactic visible radius (kpc) Relation (18)	Galactic effective radius (kpc) Relation (17)	Geometric radius (kpc) Relation (19)	Approximate core radius (kpc) Relation (21)
Dwarf (LSB & HSB) Galaxies								
DDO 154	2.6E+39	2.9	74.1	42.4	3.1	12.1	6.1	0.5
F583-4	7.6E+39	4.9	83.0	61.7	4.3	25.5	10.5	0.7
DDO 170	8E+39	5.0	83.4	62.8	4.4	26.4	10.8	0.7
DDO 168	8.4E+39	5.1	83.7	63.9	4.5	27.3	11.0	0.7
UGC 2259	1.54E+40	7.0	87.4	79.3	5.3	42.2	14.9	0.8
NGC 3109	1.56E+40	7.0	87.5	79.7	5.3	42.6	15.0	0.8
NGC 1560	1.58E+40	7.1	87.6	80.1	5.3	42.9	15.1	0.8
UGC 6446	1.66E+40	7.2	87.9	81.5	5.4	44.5	15.5	0.9
UGC 7089	1.72E+40	7.4	88.0	82.6	5.5	45.6	15.8	0.9
UGC 6923	1.92E+40	7.8	88.6	85.9	5.6	49.4	16.7	0.9
NGC 4096	2.14E+40	8.2	89.1	89.3	5.8	53.4	17.6	0.9
NGC 55	2.34E+40	8.6	89.6	92.3	5.9	57.0	18.4	0.9
NGC 5585	2.34E+40	8.6	89.6	92.3	5.9	57.0	18.4	0.9
UGC 6818	2.62E+40	9.1	90.1	96.1	6.1	61.9	19.5	1.0
UGC 6399	2.68E+40	9.2	90.2	96.9	6.2	62.9	19.7	1.0
UGC 6917	4.12E+40	11.4	91.9	113.3	6.9	86.0	24.4	1.1
UGC 3691	5.66E+40	13.4	93.0	127.3	7.6	108.5	28.6	1.2
NGC 4062	5.96E+40	13.7	93.2	129.7	7.7	112.7	29.4	1.2
NGC 3972	8.18E+40	16.1	94.1	145.7	8.3	142.2	34.4	1.3
NGC 4389	8.8E+40	16.7	94.3	149.7	8.5	150.0	35.7	1.4
NGC 4085	1.02E+41	17.9	94.7	158.1	8.8	167.5	38.5	1.4
NGC 4569	1.25E+41	19.8	95.2	170.1	9.3	193.8	42.5	1.5
NGC 3949	1.3E+41	20.3	95.3	172.9	9.4	200.2	43.4	1.5
NGC 3877	1.73E+41	23.4	95.9	192.1	10.1	247.0	50.1	1.6
NGC 2708	1.89E+41	24.4	96.1	198.3	10.4	263.3	52.3	1.7

Galaxy Name	Galaxy visible mass (kg)	Dark mass factor X_{dark}	Dark mass%	Estimated rotation speed (km/sec) Relation (13)	Galactic visible radius (kpc) Relation (18)	Galactic effective radius (kpc) Relation (17)	Geometric radius (kpc) Relation (19)	Approximate core radius (kpc) Relation (21)
LSB Galaxies								
UGC 6446	1.66E+40	7.2	87.9	81.5	5.4	44.5	15.5	0.9
F583-1	3.12E+40	9.9	90.8	102.4	6.4	70.2	21.3	1.0
NGC 1003	3.28E+40	10.2	91.0	104.3	6.5	72.8	21.8	1.0
NGC 598	3.56E+40	10.6	91.4	107.4	6.7	77.3	22.7	1.1
NGC 4183	4.08E+40	11.3	91.9	112.9	6.9	85.4	24.3	1.1
UGC 6983	4.24E+40	11.6	92.0	114.5	7.0	87.8	24.8	1.1
UGC 6930	4.34E+40	11.7	92.1	115.5	7.0	89.3	25.1	1.1
F563-1	4.52E+40	11.9	92.3	117.2	7.1	92.0	25.6	1.1
NGC 247	4.54E+40	12.0	92.3	117.4	7.1	92.3	25.6	1.1
F568-3	6.16E+40	13.9	93.3	131.3	7.7	115.4	29.9	1.2
NGC 3495	8.32E+40	16.2	94.2	146.6	8.4	144.0	34.7	1.3
F571-8	1.09E+41	18.6	94.9	162.0	9.0	175.9	39.8	1.4
NGC 4010	1.11E+41	18.7	94.9	163.1	9.0	178.2	40.1	1.4
NGC 3917	1.25E+41	19.8	95.2	170.3	9.3	194.3	42.6	1.5
UGC 6614	2.27E+41	26.8	96.4	212.4	10.9	302.3	57.4	1.7
NGC 3672	2.97E+41	30.6	96.8	234.7	11.7	368.9	65.6	1.9
NGC 1417	3.32E+41	32.3	97.0	244.5	12.0	400.5	69.4	1.9
HSB Galaxies								
NGC 3034	1.04E+40	5.7	85.1	68.9	4.7	31.8	12.3	0.8
NGC 4448	3.96E+40	11.2	91.8	111.7	6.9	83.6	24.0	1.1
NGC 6503	3.96E+40	11.2	91.8	111.7	6.9	83.6	24.0	1.1
NGC 300	4.06E+40	11.3	91.9	112.7	6.9	85.1	24.3	1.1
NGC 3769	5.18E+40	12.8	92.7	123.2	7.4	101.7	27.4	1.2
NGC 4303	6.16E+40	13.9	93.3	131.3	7.7	115.4	29.9	1.2
NGC 4736	6.3E+40	14.1	93.4	132.4	7.8	117.4	30.2	1.2
NGC 660	6.4E+40	14.2	93.4	133.1	7.8	118.7	30.5	1.2
NGC 2403	7.6E+40	15.5	93.9	141.8	8.2	134.7	33.2	1.3
NGC 1808	8.2E+40	16.1	94.1	145.8	8.3	142.4	34.5	1.3
NGC 4138	8.62E+40	16.5	94.3	148.5	8.5	147.8	35.3	1.3

Galaxy Name	**Galaxy visible mass (kg)**	**Dark mass factor** X_{dark}	**Dark mass%**	**Estimated rotation speed (km/sec) Relation (13)**	**Galactic visible radius (kpc) Relation (18)**	**Galactic effective radius (kpc) Relation (17)**	**Geometric radius (kpc) Relation (19)**	**Approximate core radius (kpc) Relation (21)**
NGC 4945	9.16E+40	17.0	94.4	151.9	8.6	154.5	36.4	1.4
NGC 5907	9.18E+40	17.0	94.4	152.0	8.6	154.8	36.5	1.4
NGC 3198	1.11E+41	18.7	94.9	163.0	9.0	178.0	40.1	1.4
NGC 4527	1.11E+41	18.7	94.9	163.0	9.0	178.0	40.1	1.4
NGC 4013	1.2E+41	19.5	95.1	167.9	9.2	188.8	41.7	1.5
NGC 4631	1.23E+41	19.7	95.2	169.3	9.3	192.0	42.2	1.5
NGC 5236	1.23E+41	19.7	95.2	169.4	9.3	192.2	42.2	1.5
NGC 6951	1.24E+41	19.8	95.2	170.0	9.3	193.6	42.5	1.5
UGC 6973	1.28E+41	20.1	95.3	171.9	9.4	198.0	43.1	1.5
NGC 253	1.39E+41	20.9	95.4	177.0	9.6	209.9	44.8	1.5
NGC 3031	1.39E+41	20.9	95.4	177.1	9.6	210.2	44.9	1.5
NGC 3379	1.4E+41	21.0	95.5	177.5	9.6	211.0	45.0	1.5
NGC 4051	1.44E+41	21.3	95.5	179.5	9.7	215.9	45.7	1.5
NGC 4258	1.46E+41	21.4	95.5	180.3	9.7	217.7	46.0	1.5
NGC 5194	1.46E+41	21.4	95.5	180.3	9.7	217.7	46.0	1.5
NGC 891	1.49E+41	21.7	95.6	181.9	9.8	221.7	46.5	1.6
NGC 3893	1.54E+41	22.0	95.7	184.0	9.8	226.7	47.2	1.6
NGC 3521	1.58E+41	22.3	95.7	185.6	9.9	230.8	47.8	1.6
IC 342	1.59E+41	22.4	95.7	186.2	9.9	232.1	48.0	1.6
NGC 5055	1.68E+41	23.0	95.8	189.8	10.1	241.3	49.3	1.6
NGC 3079	1.75E+41	23.5	95.9	192.7	10.2	248.7	50.3	1.6
NGC 6946	1.79E+41	23.8	96.0	194.5	10.2	253.4	50.9	1.6
Milky Way	1.82E+41	24.0	96.0	195.8	10.3	256.9	51.4	1.6
NGC 3628	1.83E+41	24.0	96.0	195.9	10.3	257.1	51.4	1.6
NGC 1068	1.88E+41	24.4	96.1	198.2	10.4	263.1	52.2	1.7
NGC 3726	1.92E+41	24.6	96.1	199.6	10.4	266.8	52.7	1.7
NGC 2903	1.93E+41	24.7	96.1	200.1	10.4	268.1	52.9	1.7
NGC 4088	1.95E+41	24.8	96.1	200.7	10.5	269.7	53.1	1.7
NGC 5033	1.98E+41	25.0	96.2	201.9	10.5	273.0	53.6	1.7
NGC 5457	2.04E+41	25.4	96.2	204.1	10.6	279.1	54.4	1.7

Galaxy Name	Galaxy visible mass (kg)	Dark mass factor X_{dark}	Dark mass%	Estimated rotation speed (km/sec) Relation (13)	Galactic visible radius (kpc) Relation (18)	Galactic effective radius (kpc) Relation (17)	Geometric radius (kpc) Relation (19)	Approximate core radius (kpc) Relation (21)
NGC 4100	2.06E+41	25.5	96.2	204.9	10.6	281.1	54.6	1.7
NGC 4157	2.33E+41	27.1	96.4	214.4	11.0	307.8	58.1	1.7
NGC 4217	2.58E+41	28.5	96.6	222.8	11.3	332.5	61.2	1.8
NGC 2590	2.81E+41	29.8	96.7	229.9	11.5	353.9	63.8	1.8
NGC 1365	2.99E+41	30.7	96.8	235.3	11.7	370.7	65.8	1.9
NGC 2998	3.03E+41	30.9	96.9	236.3	11.7	373.9	66.2	1.9
NGC 4565	3.62E+41	33.8	97.1	252.6	12.3	427.2	72.4	2.0
NGC 801	4.01E+41	35.6	97.3	262.4	12.6	461.1	76.3	2.0
NGC 224	4.04E+41	35.7	97.3	263.0	12.6	463.2	76.5	2.0
NGC 3953	4.09E+41	35.9	97.3	264.3	12.7	468.0	77.0	2.0
NGC 7331	4.29E+41	36.8	97.4	269.0	12.8	484.8	78.9	2.0
NGC 4321	4.33E+41	37.0	97.4	270.0	12.9	488.2	79.2	2.0
NGC 1097	4.54E+41	37.8	97.4	274.6	13.0	505.0	81.1	2.1
NGC 3992	5.03E+41	39.8	97.6	285.4	13.4	545.5	85.4	2.1
NGC 5533	5.76E+41	42.6	97.7	300.1	13.8	603.4	91.4	2.2
NGC 6674	6.5E+41	45.2	97.8	313.8	14.3	659.7	97.0	2.3
NGC 2841	6.61E+41	45.6	97.9	315.8	14.3	668.2	97.8	2.3

13) With reference to MSTG and MOND approaches, approximate galactic core radius can be expressed as,

$$r_{core} \approx \frac{r_{vis}}{2\pi} \approx \left(\frac{1}{2\pi}\right)\sqrt{\frac{GM_{vis}}{\left(X_{dark}+1\right)c\omega_0}} \qquad (21)$$

14) See Table 2 for the estimated visible, effective, geometric and core radii of galaxies. Our estimation seems to be in a right track.
15) By minimizing the errors in estimating the visible mass of galaxy, accuracy can be improved. Point to be noted is that, there is no correlation between photometric mass estimations and parametric mass estimations. Similarly, in some cases, there is no correlation between MSTG mass estimations and MOND mass estimations. It needs a careful analysis.
16) In near future, by thoroughly studying the galactic dark mass distribution and corresponding deviations, variations in flat rotation speeds can be analyzed in a systematic approach.
17) We are also working on developing alternative relations for estimating X_{dark}. On lower side, by studying the ultra faint dwarf galaxies it seems possible to fine tune X_{dark}.
18) Interesting point to be noted is that, for small galaxies whose mass is less than 3.179×10^{38} kg, their dark mass seems to be less than their visible mass. Whether it is – 'correct or not' – can be confirmed with their galactic rotational curves. For a galaxy of visible mass $10^6 M_{Sun}$, galactic flat rotation speed seems to be 5.14 km/sec. It needs further investigation with respect to least massive galaxy, Segue2. According to Evan N. Kirby et al [18]: "Either Segue 2 would be the first of a vast class of new galaxies to be discovered with very low luminosities and very low dark matter content, or it would have to represent a rare case of a dark matter halo that is typically too small to host a galaxy but, for some reason, managed to form a small number of stars over at least 100 Myr."
19) Relation (16) seems to be very simple in representation, easy to follow and simple to visualize and analyze MONDin approach connected with galactic structures and cosmic structure [4].
20) Considering "merging" of any two galaxies, based on relations (3) and (4), one can expect a considerable increase in dark mass and it can be verified with increased flat rotation speed of orbiting stars after reaching a kind of combined galactic stability. Mean while, it is better to understand the enforced reasons of merging, time scale of merging, steps involved in merging and time scale of reaching galactic stability as a whole (after merging).

9. ABOUT THE WEAK GRAVITATIONAL CONSTANT

Readers are encouraged to see the particle level applications of Weak gravitational constant proposed by Roberto Onofrio in 2013 [19]. Since it is generally believed that, dark matter is a characteristic form of weak interaction, we have taken an initiative in developing a reference mass of 3.179×10^{38} kg with weak gravitational constant [7,8,9] wide relations (1) to (7). We are working on establishing our published concepts pertaining to weak gravitational constant in various possible ways [20]. In a cosmological approach, we noticed that, with current cosmic mass and Planck mass, there is a scope for developing such a (varying) reference mass unit. We are working in this direction also.

10. CONCLUSION

Based on the data presented in Table 1 and Figs. 1-3, it is possible to conclude that, cosmic maximum angular acceleration, galactic dark mass and visible mass play a combined role in estimating galactic flat rotation speed. Proceeding further, based on relations (1) and (3) and data presented in Tables 1-2 and Figs. 1-3, it seems possible to conclude that, galactic dark matter is a representation of weakly interacting massive foam proportional to $\left(M_{vis}\right)^{\frac{3}{2}}$. We are working on understanding and estimating the proposed reference mass unit of $M_{\text{Ref}} \cong 3.179\times10^{38}$ kg in all possible ways.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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