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Statistical Analysis on the Waiting Line System in Automated Teller Machines (ATM): A Study of Fidelity Bank Plc, Service Point, Plateau State, Nigeria

Kuje Samson[1*], Abubakar, Muhammad Auwal[2], Kuje, Habila Akolo[3] and Moses, Longji Dashal[1]

[1]Department of Mathematical Science, Abubakar Tafawa Balewa University (ATBU), Bauchi, Nigeria.
[2]Department of Statistics, Nasarawa State University (NSUK), Keffi, Nigeria.
[3]Zenith Bank Plc, Kano State, Nigeria.

Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

The purpose of this paper is to find out the operating characteristics of the ATM service point of the Fidelity bank Plc, plateau state. Specifically, a computer package (MS60) was used for analyzing the data. Results obtained from the analysis showed the traffic intensity (ρ) to be 0.96, which indicated that the service facility is highly utilized. The average length of the queue was found to be 21 while the average waiting time in the queue was 1.10 hours. On the basis of this investigation, the conclusion was made that the service utility is highly utilized, implying that there are more customers than the service point can accommodate thus giving rise to the lengthy customer waiting time. It is recommended that one additional ATM be deployed to the bank's premises so as to minimize customer waiting time and to also increase the service rate.

Keywords: Automated teller machines; traffic intensity; fidelity bank; customer service.

**Corresponding author: E-mail: kujesamson@gmail.com;*

1 Introduction

By way of introduction, a common situation occurring in everyday life is that of waiting line or queuing. Waiting lines are commonly seen at bus stops, fast-food joints, ticket booths, doctors' clinics, bank counters, traffic lights etc. Waiting lines are also found in workshops where the machines wait to be repaired; at a tool crib where the mechanics wait to receive tools, in a warehouse where items wait to be used, incoming calls wait to mature in the telephone exchange, trucks wait to be unloaded, aeroplanes wait either to take off or land and so on [1].

Waiting in line is a part of our everyday life. In general, a queue is formed when either customer (human beings or physical entities) requiring service wait because the number of customers exceeds the number of service facilities, or service facilities do not work efficiently and take more time than prescribed to serve a customer Sharma [2].

Taha (2002) defined a waiting line simply as a queue.

Similarly, Jag dish Hiray [3] defined a waiting line system by two important elements: the population source of its customers and the process or service system. The customer population can be considered as finite or infinite.

Anderson [4] stated that managing the waiting line creates a great dilemma for managers seeking to improve the return on investment of their operations. Customers dislike waiting intensely. If they feel they are waiting too long at your firm for service, they will either leave the line prematurely or not return to your firm the next time they need service. This will reduce customer demand and eventually revenue and profit. Furthermore, longer waiting time increases cost because longer waiting time equals more customers in a firm’s building or facility. Hence, a firm will need more space for the customers to wait, which in itself means additional costs.

In this technology era, no where is waiting for line more vividly experienced than at the ATMs (Automatic Teller Machines). Globally, Automatic Teller Machines (ATMs) have been adopted and are still being adopted by banks. They offer considerable benefits to both banks and their customers. The machines can enable depositors to make financial transactions at more convenient times and places than during banking hours at branches. Most importantly, ATMs have been designed to provide efficient and improved services to customers at the shortest time possible. Yet, customers spend a considerable amount of time before they are finally serviced by the facility.

Waiting lines affect people every day, which is why a primary goal in many businesses is to provide the best level of service possible. Minimizing those waiting lines is a key part of creating a positive experience for the customer.

1.1 Statement of the problem

Standing in line can cause extreme boredom, annoyance and even rage to customers. Customers are often forced to wait in line whenever the service facility is busy. Although, s ATMs have been designed to provide efficient and improved services to customers at the shortest time possible, yet customers wait too long before they are finally serviced by the facility.

This is principally due to variation in arrival and service time, which eventually leads to the formation of queue. Sometimes, however, queues form because resources are limited. If the service facility is busy or occupied, a queue is formed when customers arrive more frequently than usual, or when it takes longer than usual to serve a customer. If the queue gets too long, customers may be compelled to look elsewhere for service. Nonetheless, in situations where queue exists in a system, it is only appropriate to try to minimize the length of the queue rather than attempt to eliminate it completely. Complete elimination may be impossible or even undesirable.

Therefore, a systematic study of waiting line system will assist the management of the Bank in taking certain decisions that may minimize the length of time a customer spends in a service facility.

1.2 Aim and objectives of the study

Aim: The aim of this work is to examine the waiting line at the ATM (Automatic Teller Machine) service point of the Fidelity bank Plc located in North central, plateau state by estimating the operating characteristics in order to optimize among other things, waiting time of customers and service rate of the facility.

This study tends to employ queuing theory to determine the following queuing characteristics which are usually of interest. They are:

1. Average number of customers in the waiting line.
2. Average number of customers in the system.
3. Average time a customer spends in the waiting line.
4. Average time a customer spends in the system.
5. The probability that an arriving customer has to wait.

1.3 Significance of the study

The important aspect of this study is to predict the operating characteristics of the service facility. The result from the analysis will be used to determine if it is advisable to increase the service facility or not. The information will help towards:

(a) Minimizing inconveniencies and frustrations associated with waiting.
(b) Intensifying periodic maintenance of the facility for improved service delivery.
(c) Enabling the findings to help bank management to appreciate the importance of waiting line models and to encourage the further application of these models in addressing queuing problems.

1.4 Scope of the study

The study is limited to the waiting line of customers at the ATM facility of Fidelity bank Plc plateau state.

1.5 Limitation of the study

The limitations accompanying this paper are:

(a) Limited time for data collection.
(b) The problem of recording service time for each customer.
(c) The complexity of the study.

1.6 Definition of terms

The following terms are defined based on the context of this study:

1. ATM – Automatic Teller Machines
2. Arrival time – The number of arrivals per unit of time.

3. Balk – To leave without joining the queue because the queue is too long or by estimating the excessive waiting time for the desired service [2].
4. Event – A set of the outcome.
5. Inter-arrival Time – The elapsed time between the arrivals of successive customers in a queuing system.
6. Jockey – Customers move from one queue to another hoping to receive service more quickly [2].
7. Model – An abstract, mathematical representation of a real or physical system.
8. Outcome – The fundamental result of a probabilistic experiment.
9. Queue – A waiting line.
10. Random Variable – A symbolic representation of an outcome.
11. Renege – A waiting customer leaves the queue due to certain reasons [2].
12. Sample Space – The set, collection, of all possible outcomes.
13. Service Rate – The number of customers served per unit of time.
14. Service Time –The time a customer spends receiving service in a queuing system.
15. Steady State –A condition representative of a systems long-run behaviour; for example, an assembly line starting without parts in the process will be in a transient state until such time that the various stations are being utilized at approximately their expected levels, at which point steady-state has been achieved.
16. Stochastic process – A process organized into states in which movement from state to state is governed by probabilities; examples include the number of customers in a queuing system and levels of inventory on hand.
17. Waiting time – The elapsed time between a customer's arrival and the beginning of service [5].

1.7 Queue characteristics

Nosek and Wilson (2001) stated that the queuing system can be characterized by four components or four main elements. These are: the arrival, the queue discipline, the service mechanism and the cost structure. Taha (1976) on the other hand stated that queuing systems are characterized by five (5) components. The arrival pattern of customers, the service pattern, the number of servers, the capacity of the facility to hold customers, and the order in which the customers are served.

2 The Arrival Pattern of Customers

The arrival is the way in which a customer arrives and enters the system for services. It is the system input process. It is how units (customer) joined the queues.

Whenever customers arrive at a rate that exceeds the processing system rate, a queue will be formed. The arrival process of customers is usually specified by the inter-arrival time (the time between successive customer arrivals to the service facility). It may be deterministic (known exactly) or it may be a random variable whose probability distribution is presumed known. The arrival process can be;

- Regular arrival; that is it follows a Poisson distribution with average arrival rate λ. It may be
- In a completely random manner
- Singly or in batches
- Non-stationary arrival

2.1 The queue discipline

Taha (1976) defined queuing discipline as the order in which customers are served. It is the rule for determining the formation of the line or queue and the order in which jobs are processed or order in which customers are served. For this research work, the customers are the clients demanding service at the ATM service point of Fidelity bank, Borno.

These could be on:

- First in first out (FIFO) basis (i.e. serve in the order of arrival)
- Last in first out (LIFO) basis (i.e. the customer who arrives last is the next served)
- First in last served
- A random basis
- Priority basis

Also, there are other factors of customers' behaviour such as bulking, reneging, and jockeying that require considerations as well.

2.2 The service mechanism

Adam et al. [6] viewed that the uncertainty involved in the service mechanism is the number of servers, the number of customers getting served at any time, and the duration and mode of service. Networks of queues consist of more than one server arranged in series or parallel. Random variables are used to represent service time, and the number of servers when appropriate.

2.3 The service pattern

The service pattern is usually specified by the service time (the time required by one server to completely serve one customer). The service time may be deterministic or it may be a random variable whose probability distribution is presumed known. It may depend on the number of customers already in the facility or it may be state independent [1]. Also of interest is whether a customer is attended to completely by one server or the customer requires a sequence of servers. Unless stated to the contrary, the standard assumption will be that one server can completely serve a customer.

2.4 System capacity

Taha (1976) stated that the system capacity is the maximum number of customers, both those in service and those in the queue(s), permitted in the service facility at the same time.

He explained further that whenever a customer arrives at a facility that is full, the arriving customer is denied entrance to the facility. Such a customer is not allowed to wait outside the facility (since that effectively increases the capacity) but is forced to leave without receiving service. A system that has no limit on the number of customers permitted inside the facility has infinite capacity, while a system with a limit has a finite capacity.

2.5 Calling population

This is a set of potential customers expected to receive the services. In this research work, the inter-arrival time of the customers follows an exponential distribution. The number of arrivals over a specific time interval follows a Poisson distribution with mean λ. That is,

$P_n = \lambda^n e^{-\lambda}/n!$ $\quad\quad$ n= 0,1,2,……..

2.6 Research design

The purpose of this paper is to examine the performance characteristics of the Fidelity bank Plc. Western, Maiduguri ATM service point. The system's characteristics of interest that will be examined in this research work include; number of arrivals (number of customers arriving to the service point at a given time), service time (the time it takes for one server to complete customer's service), the average number of customers in the system, and the average time a customer spends in the system.

The results of the operating characteristics will be used to evaluate the performance of the service mechanism and to ascertain whether customers are satisfied with the banks' services. This is essential since a customer's experience of waiting can radically influence his/her perception of the service quality of the bank.

2.7 Model construction

Gupta and Hira (2007) defined a model as an idealized representation of the real-life situation.

In order to keep the model as simple as possible, however, some assumptions need to be made.

2.7.1 Assumptions made on the system

1. Single channel queue.
2. There is an infinite population from which customers originate.
3. Poisson arrival (Random arrivals).
4. Exponential distribution of service time.
5. Arrival in the group at the same time (i.e. bulk arrival) is treated as single arrival.
6. The waiting area for customers is adequate.
7. The queue discipline is First Come First Served (FCFS).

Thus, the model considered in this study is the single server model of queuing systems.

2.7.2 M/M/1 systems

An M/M/1:(∞/FCFS) system is a queuing system having exponentially distributed inter-arrival times with parameter (λ); exponentially distributed service times with parameter (μ); one service; no limit on the system capacity; and a queue discipline of first come first served. The constant (λ) is the average customer arrival rate; the constant (μ) is the average service rate of customer. Both are in units of customers per unit time.

The expected inter-arrival time and the expected time to serve one customer are ($1/\lambda$) and ($1/\mu$) respectively.

An M/M/1 system is a Poisson birth-death process. The probability, $P_n(t)$ i.e. the system has exactly n customers either waiting for service or in service at time t satisfies the Kolmongorov equation with $\lambda_n = \lambda$ and $\mu_n = \mu$, for all n.

The steady state probabilities for a queuing system are;

$P_n = \text{Lim } P_n(t)$ as $t \to \infty$ $\quad (n = 0, 1, 2, 3, \ldots)$

If the limit exist.

For an M/M/1 system, we define utilization factor (traffic intensity) as $\rho = \lambda/\mu$ and steady-state probabilities as $P_n = \rho^n(1-\rho)$

if $P<1$.

But if $P>1$, the arrival comes at a faster rate than the server can accommodate.

2.7.3 Measure of effectiveness

L= the average number of customers in the system
Lq= the average length of the queue

W= the average time a customer spends in the system
Wq = the average time a customer spends in the queue
W(t) = the probability that a customer spends more than t units of time in the system.
Wq(t) = the probability that a customer spends more than t unit of time in the queue.

For an M/M/1 system $\hat{\lambda} = \lambda$ and the six (6) measures are explicitly:

$$L = \frac{\rho}{(1-\rho)}$$

$$Lq = \rho^2/(1-\rho)$$

$$W = \frac{1}{(\mu-\lambda)}$$

$$Wq = \frac{\rho}{(\mu-\lambda)}$$

$$W(t) = e^{-t/w} \quad (t \geq 0)$$

$$Wq(t) = \rho e^{-t/w} \quad (t \geq 0)$$

2.8 Research instruments

The data will be collected with the help of a wristwatch (for recording the inter-arrival time), a stopwatch (for recording service time), and writing materials for the research work.

2.9 Method of data collection

The data for this study has been collected from the primary source and is limited to the ATM service point of Fidelity bank Plc located in North central, plateau (Appendix A). Data will be collected by observation, in which the number of customers arriving at the facility will be recorded, as well as each customer's arrival and service time respectively. The assistance of a colleague will be sought in recording the service time while the researcher records arrival time. The period for the data collection will be during busy working hours (i.e. 8:00am to 4:00pm) and for a period of ten (10) working days.

2.10 Method of data analysis

Based on the system's arrival and service pattern, and the assumptions made during data collection, the M/M/1 queuing system will be used to analyze the data collected using an MS60 computer package.

3 Analysis and Interpretation of Results

The Tables below show a summary of frequencies for the inter arrival time and service time from the data collected as depicted in appendix B and C.

Table 1. Frequencies for inter arrival time

X	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11
Y	94	89	94	23	15	10	4	5	2	0	2

At X=0-1 implies that 94 times, customers arrived at an inter arrival time between 0 to 1 minute

Table 2. Frequencies for service time

P	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9
Q	23	171	91	17	16	10	6	0	3

At P=0-1 implies that 23 times, customers were served at a period of not more than 1 minute

3.1 Arrival rate

The arrival rate is given by $\frac{1}{\lambda} = \frac{\sum_{i=1}^{11} XiYi}{\sum_{i=1}^{11} Yi}$

Where $\sum_{i=1}^{11} XiYi$ =1042, $\sum_{i=1}^{11} Yi$=338

Therefore, $\frac{1}{\lambda} = \frac{1042}{338}$=3.0832

Thus, the Average arrival rate per hour

$\lambda = \frac{60}{3.0832}$ =19.46 per hour

3.2 Service rate

The average service rate is given by $\frac{1}{\mu} = \frac{\sum_{j=1}^{9} PjQj}{\sum_{j=1}^{9} Qj}$

Where $\sum_{j=1}^{9} PjQj$ =1012, $\sum_{j=1}^{9} Qj$ =343

$\frac{1}{\mu} = \frac{1012}{343}$=2.9513

The average service rate per hour

$\mu = \frac{60}{2.9513}$=20.33

3.3 Traffic intensity and measures of effectiveness

The traffic intensity and measures of effectiveness are calculated using an MS60 software package.

The output is given below:

Summary of a one channel waiting line with

Mean number of arrivals λ=19.46

Mean number of services μ=20.33

L=	22.3678	Lq=	21.4106
W=	1.1494	Wq=	1.1002
W(t)=	0.0428	Wq(t)=	0.9572

From the results above,

The average number of customers in the system, L =22
The average length of the queue, Lq = 21
The average time a customer spends in the system, W = 1.1494
The average time a customer spends in the queue, Wq = 1.1002

The probability that a customer spends more than t units of time in the system, W(t) = 0.0428

The probability that a customer spends more than t unit of time in the queue, Wq(t) = 0.9572

Results from the study, it revealed that the queue is quite long and as such customers that come to the ATM would have to wait too long before they are serviced by the ATM. Observations also showed that due to network complexities the service rate is relatively slow.

While this may seem to confirm the fact that the excessively long queue and lengthy service time could be due to the influx of customers and shortage of service mechanism, the opinion of management regarding the inadequacy of the service mechanism in meeting the needs of customers was sought.

Asked if management would be willing to reduce the waiting time of customers by deploying more ATM's to the banks' premises, they answered in the affirmative. The bank, however, added that it could only afford one additional ATM for the time being.

Thus, with the bank agreeing to deploy one additional service point, and considering a service pattern of a single queue, multiple servers in parallel, the output of the M/M/S :(∞/FCFS) where S=2 is given as:

Summary of a two channel waiting line with

Mean number of arrivals

$\lambda=19.46$

4 Summary, Recommendations and Conclusion

Results from the analysis showed that the traffic intensity (ρ) to be 0.96. Since we obtained the value of the traffic intensity, otherwise known as the utilization factor to be less the one (i.e. $\rho<1$), it could be concluded that the system operates under steady-state condition. Thus, the value of the traffic intensity, which is the probability that the system is busy, implies that 95% of the time period considered during data collection the system was busy as against 4% idle time. This indicates high utilization of the system.

Results from the MS60 software package on the measures of effectiveness for an M/M/1:(∞/FCFS) shows the average number of customers in the system to be 22, the average number of customers in the queue is 21.

A customer spends an average of 1.15 hours before he/she is serviced by the system while a customer is likely to spend an average of 1.10 hours in the queue waiting for service.

Comparatively, the measures of effectiveness for an M/M/2:(∞/FCFS) puts the traffic intensity at 45%, the average number of customers in the system at approximately 5 while the average number of customers in the queue is 4. Similarly, a customer spends an average of 0.063 hours in the system while queue waiting time for a customer is 0.014 hours on the average.

4.1 Recommendations

Hitchin (1992) in his study on waiting lines highlighted that to contain queue length, utilization (i.e. traffic intensity) must be less than one, the server must have unused capacity and the server must at times be sitting idle. An average of one entity is not uncommon per queue. This also corresponds to 50% channel utilization.

1. The findings of this study have revealed that the system is highly, if not over-utilized. This implies that arrival comes at a faster rate than the system can accommodate. The following recommendations if accepted and implemented by the bank management may help in tackling these problem. The need for the management of the bank to deploy another ATM (i.e. an M/M/S with S=2) within the bank's premises as this will minimize the waiting time of customers and hence reducing the inconveniences and frustrations associated with waiting.
2. The bank should review its maintenance policy so as provide timely and periodic maintenance on these machines. This will drastically reduce machine or server complexities while at the same time increasing service efficiency.

4.2 Conclusion

According to Beasley [7], in designing queuing systems we need to aim for a balance between service to customers (short queues implying many servers) and economic considerations (not too many servers). Though the provision of an additional service mechanism may be capital intensive, it would pay the bank more since the primary aim of every business organization besides profit-making is customer satisfaction.

The conclusion was reached without considering cost models for the system (i.e. the cost of deploying an additional ATM and cost implication resulting from the banks' inability to provide additional service point). However, the investigator strongly recommends that management should make provision for one additional ATM so as to enable her minimizes customer waiting time and improve service rate.

Disclaimer

The products used for this research are commonly and predominantly use products in our area of research and country. There is absolutely no conflict of interest between the authors and producers of the products because we do not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was not funded by the producing company rather it was funded by the personal efforts of the authors.

Competing Interests

Authors have declared that no competing interests exist.

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APPENDIX A

Table A1. Data collected on 20th May, 2019 from the ATM service point of the Fidelity Bank Plc, North Central

NA	ART	SRT	NA	ART	SRT	NA	ART	SRT
1	1	1.08	27	57	2.14	53	125	2.09
2	2	2.31	28	59	3.14	54	128	2.13
3	4	1.42	29	60	1.51	55	129	1.37
4	5	1.56	30	61	2.10	56	132	1.38
5	8	2.14	31	63	2.20	57	134	1.47
6	9	1.34	32	65	3.20	58	137	2.56
7	11	2.51	33	68	1.03	59	138	3.13
8	13	3.28	34	70	8.14	60	141	1.36
9	17	2.17	35	71	1.53	61	144	1.29
10	19	2.18	36	74	2.02	62	147	1.30
11	20	2.30	37	75	3.16	63	150	3.04
12	27	1.33	38	77	1.38	64	151	1.11
13	28	1.21	39	80	2.13	65	154	1.16
14	29	6.12	40	91	1.18	66	156	2.15
15	31	1.03	41	94	4.10	67	161	1.28
16	34	1.07	42	95	2.17	68	163	1.41
17	37	1.23	43	98	2.49	69	166	2.38
18	38	1.51	44	109	1.56	70	169	1.23
19	42	3.46	45	112	2.03	71	171	1.09
20	43	1.18	46	115	1.49	72	172	1.33
21	46	2.03	47	116	3.08	73	175	1.18
22	47	2.18	48	119	4.15	74	179	1.36
23	49	4.0	49	120	2.18			
24	50	2.35	50	121	3.17			
25	53	1.36	51	122	1.12			
26	54	1.18	52	123	1.26			

Let NA= Number of Arrivals, ART= Arrival Time (minute), SRT= Service Time (minute)

Table A2. Data collected on 21st May, 2019 from the ATM service point of the Fidelity Bank Plc, North Central

NA	ART	SRT	NA	ART	SRT	NA	ART	SRT
1	1	2.12	32	73	1.30	63	138	1.11
2	3	3.13	33	74	1.10	64	140	1.15
3	4	2.50	34	76	1.06	65	141	1.50
4	5	1.51	35	79	1.45	66	142	1.30
5	6	1.43	36	81	2.01	67	145	1.06
6	11	1.23	37	84	1.21	68	149	1.15
7	12	2.40	38	85	1.43	69	151	2.13
8	15	1.20	39	86	1.21	70	152	1.31
9	17	1.01	40	88	1.03	71	154	2.12
10	19	2.11	41	91	1.50	72	157	1.28
11	24	1.20	42	92	1.45	73	163	3.08
12	25	1.45	43	94	6.30	74	164	2.14
13	26	1.54	44	97	1.30	75	166	1.13
14	28	1.20	45	99	2.30	76	169	1.36
15	31	3.31	46	101	1.10	77	171	2.00
16	34	2.22	47	102	1.35	78	172	1.36
17	36	0.51	48	104	1.45	79	173	1.27
18	39	3.11	49	107	1.12	80	175	1.00
19	40	1.10	50	111	1.32	81	176	1.51
20	42	1.02	51	116	1.37			
21	44	1.11	52	117	1.11			
22	45	2.33	53	118	1.28			
23	48	1.55	54	121	3.12			
24	51	1.30	55	123	1.32			

Let NA= Number of Arrivals, ART= Arrival Time (minute), SRT= Service Time (minute)

Table A3. Data collected on 22nd May, 2019 from the ATM service point of Fidelity Bank Plc, North Central

NA	ART	SRT	NA	ART	SRT	NA	ART	SRT
1	1	1.23	30	67	1.30	59	157	0.43
2	2	1.1	31	69	1.23	60	161	1.33
3	4	2.36	32	72	1.17	61	166	1.13
4	5	1.50	33	73	1.15	62	167	1.30
5	8	0.41	34	75	0.30	63	170	1.40
6	10	1.23	35	78	2.15	64	172	2.31
7	11	1.37	36	79	1.30	65	175	2.44
8	13	1.32	37	83	1.51	66	177	2.13
9	15	1.41	38	85	1.48	67	180	1.00
10	17	1.36	39	88	1.50			
11	18	1.30	40	91	2.12			
12	19	2.01	41	94	1-32			
13	21	1.14	42	99	1.41			
14	23	6.17	43	105	1.42			
15	28	2.14	44	112	1.08			
16	30	1.13	45	116	1.02			

NA	ART	SRT	NA	ART	SRT	NA	ART	SRT
17	31	2.30	46	117	1.30			
18	34	1.16	47	121	1.15			
19	37	8.08	48	124	1.20			
20	39	1.20	49	128	1.45			
21	41	1.14	50	130	1.15			
22	43	1.11	51	131	1.17			
23	45	2.19	52	137	2.01			
24	50	1.17	53	140	1.16			
25	58	1.16	54	146	1.45			
26	59	1.43	55	149	1.51			
27	60	1.41	56	151	1.06			
28	63	1.42	57	152	1.14			
29	65	1.29	58	154	1.20			

Let NA= Number of Arrivals, ART= Arrival Time (minute), SRT= Service Time (minute)

Table A4. Data collected on the 23rd May, 2019 from the ATM service point of Fidelity Bank Plc, North central

NA	ART	SRT	NA	ART	SRT	NA	ART	SRT
1	1	2.13	29	84	1.30	57	161	1.42
2	4	1.27	30	87	1.33	58	164	1.00
3	5	1.20	31	89	1.42	59	165	1.01
4	7	2.42	32	91	1.03	60	166	1.37
5	11	1.23	33	92	1.18	61	167	2.58
6	13	4.18	34	93	1.10	62	171	1.15
7	21	6.31	35	96	1.30	63	173	2.35
8	30	1.42	36	99	2.11	64	176	2.16
9	31	1.03	37	102	2.14	65	180	2.23
10	32	1.19	38	105	6.09			
11	35	2.46	39	114	1.19			
12	38	1.23	40	115	1.23			
13	41	1.33	41	118	1.37			
14	42	1.33	41	120	1.46			
15	45	2.07	43	121	1.32			
16	47	1.34	44	124	1.17			
17	49	1.17	45	127	2.00			
18	50	1.32	46	130	1.00			
19	53	1.21	47	131	1.42			
20	55	2.30	48	134	1.16			
21	58	1.34	49	136	3.04			
22	66	1.46	50	141	1.07			
23	68	1.02	51	142	1.41			
24	71	1.32	52	144	2.39			
25	72	2.08	53	149	1.36			
26	76	1.13	54	51	2.12			
27	78	2.32	55	154	1.45			
28	79	1.41	56	155	4.08			

Let NA= Number of Arrivals, ART= Arrival Time (minute), SRT= Service Time (minute)

Table A5. Data collected on 24th May, 2019 from the ATM service point of Fidelity Bank Plc, North Central

NA	SRT	AST	NA	SRT	AST
1	1.18	1	29	2.16	2
2	1.51	2	30	1.21	1
3	2.00	2	31	1.32	1
4	1.32	1	32	1.50	2
5	1.08	1	33	1.14	1
6	1.19	1	34	1.16	1
7	1.16	1	35	3.30	3
8	2.12	2	36	2.00	2
9	1.33	1	37	1.14	1
10	1.30	1	38	1.32	1
11	1.42	2	39	1.52	2
12	1.14	1	40	2.08	2
13	2.36	2	41	1.0	1
14	1.21	1	42	2.07	2
15	2.31	2	43	1.03	1
16	1.50	2	44	1.30	1
17	1.13	1	45	2.23	2
18	1.24	1	46	1.32	1
19	1.32	1	47	1.41	2
20	2.22	2	48	1.01	1
21	2.16	2	49	1.10	1
22	2.14	2	50	2.41	3
23	1.31	1	51	2.32	2
24	2.36	2	52	1.13	1
25	2.44	3	53	2.14	2.14
26	4.32	4	54	3.25	3.25
27	1.16	1	55	3.16	3.16
28	1.07	1	56	5.47	5.47

Let NA= Number of Arrivals, ART= Arrival Time (minute), SRT= Service Time (minute)

APPENDIX B

Table B1. Arrival time and inter arrival time for data collected on 20th May, 2013

NA	ART	INT	NA	ART	INT	NA	ART	INT
1	1	-	27	57	3	53	125	2
2	2	1	28	59	2	54	128	3
3	4	2	29	60	1	55	129	1
4	5	1	30	61	1	56	132	3
5	8	3	31	63	2	57	134	2
6	9	1	32	65	2	58	137	3
7	11	2	33	68	3	59	138	1
8	13	2	34	70	2	60	141	3
9	17	4	35	71	1	61	144	3
10	18	1	36	74	3	62	147	3
11	20	2	37	75	1	63	150	3
12	27	7	38	77	2	64	151	1

NA	ART	INT	NA	ART	INT	NA	ART	INT
13	28	1	39	80	3	65	154	3
14	29	1	40	91	11	66	156	2
15	31	2	41	94	3	67	161	5
16	34	3	42	95	1	68	163	2
17	37	3	43	98	3	69	166	3
18	38	1	44	109	11	70	169	3
19	42	4	45	112	3	71	171	2
20	43	1	46	115	3	72	172	1
21	46	3	47	116	1	73	175	3
22	47	1	48	119	3	74	179	4
23	49	2	49	120	1			
24	50	1	50	121	1			
25	53	3	51	122	1			
26	54	1	52	123	1			

Let NA= Number of Arrivals, ART=Arrival Time (minute), INT= Inter Arrival Time (minute)

Table B2. Arrival time and inter arrival time for data collected on 21st May, 2019

NA	ART	INT	NA	ART	INT	NA	ART	INT
1	1	-	29	61	1	58	128	1
2	3	2	30	69	8	59	130	2
3	4	1	31	72	3	60	131	1
4	5	1	32	73	1	61	134	3
5	6	1	33	74	1	62	136	2
6	11	5	34	76	2	63	138	2
7	12	1	35	79	3	64	140	2
8	15	3	36	81	2	65	141	1
9	17	2	37	84	3	66	142	1
10	19	2	38	85	1	67	145	3
11	24	5	39	86	1	68	149	4
12	25	1	40	88	2	69	151	2
13	26	1	41	91	3	70	152	1
14	28	2	42	92	1	71	154	2
15	31	3	43	94	2	72	157	3
16	34	3	44	97	3	73	163	6
17	36	2	45	99	2	74	164	1
18	39	3	46	101	2	75	166	2
19	40	1	47	102	1	76	169	3
20	42	2	48	104	2	77	171	2
21	44	2	49	107	3	78	172	1
22	45	1	50	111	4	79	173	1
23	48	3	51	116	5	80	175	2
24	51	3	52	117	1	81	176	1
25	55	4	53	118	1			
26	58	3	54	121	3			
27	59	1	55	123	2			
28	60	1	56	125	2			
29			57	127	2			

Let NA= Number of Arrivals, ART=Arrival Time (minute), INT= Inter Arrival Time (minute)

Table B3. Arrival time and inter arrival time for data collected on 22nd May, 2019

NA	ART	INT	NA	ART	INT	NA	ART	INT
1	1	-	30	67	2	59	157	3
2	2	1	31	69	2	60	161	4
3	4	2	32	72	3	61	166	5
4	5	1	33	73	1	62	167	1
5	8	3	34	75	2	63	170	3
6	10	2	35	78	3	64	172	2
7	11	1	36	79	1	65	175	3
8	13	2	37	83	4	66	177	2
9	15	2	38	85	2	67	180	3
10	17	2	39	88	3			
11	18	1	40	91	3			
12	19	1	41	94	3			
13	21	2	42	99	5			
14	23	2	43	105	6			
15	28	5	44	112	7			
16	30	3	45	116	4			
17	31	1	46	117	1			
18	34	3	47	121	4			
19	37	3	48	124	3			
20	39	2	49	128	4			
21	41	2	50	130	2			
22	43	2	51	131	1			
23	45	2	52	137	6			
24	50	5	53	140	3			
25	58	8	54	146	6			
26	59	1	55	149	3			
27	60	1	56	151	2			
28	63	3	57	152	1			
29	65	2	58	154	2			

Let NA= Number of Arrivals, ART=Arrival Time (minute), INT= Inter Arrival Time (minute)

Table B4. Arrival time and inter arrival time for data collected on 23rd May, 2019

NA	ART	INT	NA	ART	INT	NA	ART	INT
1	1	-	30	87	3	59	165	1
2	4	3	31	89	2	60	166	1
3	5	1	32	91	2	61	167	1
4	7	2	33	92	1	62	171	4
5	11	4	34	93	1	63	173	2
6	13	2	35	96	3	64	176	3
7	21	8	36	99	3	65	180	4
8	30	9	37	102	3			
9	31	1	38	105	3			
10	32	1	39	114	9			
11	35	3	40	115	1			
12	38	3	41	118	3			
13	41	3	42	120	2			
14	42	1	43	121	1			

NA	ART	INT	NA	ART	INT	NA	ART	INT
15	45	3	44	124	3			
16	47	2	45	127	3			
17	49	2	46	130	3			
18	50	1	47	131	1			
19	53	3	48	134	3			
20	55	2	49	136	2			
21	58	3	50	141	5			
22	66	8	51	142	1			
23	68	2	52	144	2			
24	71	3	53	149	5			
25	72	1	54	151	2			
26	76	4	55	154	3			
27	78	2	56	155	1			
28	79	1	57	161	6			
29	84	5	58	164	3			

Let NA= Number of Arrivals, ART=Arrival Time (minute), INT= Inter Arrival Time (minute)

Table B5. Arrival time and inter arrival time for data collected 0n 24th May, 2019

NA	ART	INT	NA	ART	INT
1	1	-	30	87	4
2	3	2	31	89	2
3	5	2	32	91	2
4	8	3	33	92	1
5	11	3	34	97	5
6	14	3	35	104	7
7	17	3	36	112	8
8	18	1	37	114	2
9	21	3	38	115	1
10	24	3	39	118	3
11	25	1	40	121	3
12	26	1	41	126	5
13	27	1	42	127	1
14	30	3	43	131	4
15	31	1	44	133	2
16	36	5	45	135	2
17	39	3	46	139	4
18	41	2	47	142	3
19	42	1	48	144	2
20	44	2	49	148	4
21	48	4	50	150	2
22	55	7	51	154	4
23	61	6	52	159	5
24	64	3	53	163	4
25	68	4	54	166	3
26	74	6	55	172	6
27	80	6	56	178	6
28	81	1			
29	83	2			

Let NA= Number of Arrivals, ART=Arrival Time (minute), INT= Inter Arrival Time (minute)

APPENDIX C

Table C1. Service time and approximated service time for data collected on 20th May, 2019

NA	SRT	AST	NA	SRT	AST	NA	SRT	AST
1	1.08	1	27	2.14	2	53	2.09	2
2	2.31	2	28	3.14	3	54	2.13	2
3	1.42	2	29	1.51	2	55	1.37	1
4	1.56	2	30	2.10	2	56	1.38	1
5	2.14	2	31	2.20	2	57	1.47	2
6	1.34	1	32	3.20	3	58	2.56	3
7	2.51	3	33	1.03	1	59	3.13	3
8	3.28	3	34	8.14	8	60	1.36	1
9	2.17	2	35	1.53	2	61	1.29	1
10	2.18	2	36	2.02	2	62	1.30	1
11	2.30	2	37	3.16	3	63	3.04	3
12	1.33	1	38	1.38	1	64	1.11	1
13	1.21	1	39	2.13	2	65	1.16	1
14	6.12	6	40	1.18	1	66	2.15	2
15	1.03	1	41	4.10	4	67	1.28	1
16	1.07	1	42	2.17	2	68	1.41	2
17	1.23	1	43	2.49	3	69	2.38	2
18	1.51	2	44	1.56	2	70	1.23	1
19	3.46	4	45	2.03	2	71	1.09	1
20	1.18	1	46	1.49	2	72	1.33	1
21	2.03	2	47	3.08	3	73	1.18	1
22	2.18	2	48	4.15	4	74	1.36	1
23	4.08	4	49	2.18	2			
24	2.35	2	50	3.17	3			
25	1.36	1	51	1.12	1			
26	1.18	1	52	1.26	1			

Let NA= Number of Arrivals, SRT= Service Time (in minute), AST=Approximated Service Time

Table C2. Service time and approximated service time for data collected on 21st May, 2019

NA	SRT	AST	NA	SRT	AST	NA	SRT	AST
1	2.12	2	32	1.30	1	63	1.11	1
2	3.13	3	33	1.10	1	64	1.15	1
3	2.50	3	34	1.06	1	65	1.50	2
4	1.51	2	35	1.45	2	66	1.30	1
5	1.43	2	36	2.01	2	67	1.06	1
6	1.23	1	37	1.21	1	68	1.15	1
7	2.40	3	38	1.43	2	69	2.13	2
8	1.20	1	39	1.21	1	70	1.31	1
9	1.01	1	40	1.03	1	71	2.12	2
10	2.11	2	41	1.50	2	72	1.28	1
11	1.20	1	42	1.45	2	73	3.08	3
12	1.45	2	43	6.30	6	74	2.14	2
13	1.54	2	44	1.30	1	75	1.13	1
14	1.20	1	45	2.30	2	76	1.36	1
15	3.31	3	46	1.10	1	77	2.00	2
16	2.22	2	47	1.35	1	78	1.36	1

NA	SRT	AST	NA	SRT	AST	NA	SRT	AST
17	1.51	2	48	1.45	2	79	1.27	1
18	3.11	3	49	1.12	1	80	1.00	1
19	1.10	1	50	1.32	1	81	1.51	2
20	1.02	1	51	1.37	1			
21	1.11	1	52	1.11	1			
22	2.33	2	53	1.28	1			
23	1.55	2	54	3.12	3			
24	1.30	1	55	1.32	1			
25	1.45	2	56	6.28	6			
26	8.11	8	57	2.33	2			
27	2.51	3	58	1.18	1			
28	1.45	2	59	1.31	1			
29	1.01	1	60	1.42	2			
30	1.03	1	61	1.11	1			
31	1.15	1	62	1.16	1			

Let NA= Number of Arrivals, SRT= Service Time (in minute), AST=Approximated Service Time

Table C3. Service time and approximated service time for data collected on 22nd May, 2019

NA	SRT	AST	NA	SRT	AST	NA	SRT	AST
1	1.23	1	30	1.30	1	59	1.43	2
2	1.13	1	31	1.23	1	60	1.33	1
3	2.36	2	32	1.17	1	61	1.13	1
4	1.50	2	33	1.15	1	62	1.30	1
5	1.41	2	34	1.30	1	63	1.40	2
6	1.23	1	35	2.15	2	64	2.31	2
7	1.37	1	36	1.30	1	65	2.44	3
8	1.32	1	37	1.51	2	66	2.13	2
9	1.41	2	38	1.48	2	67	1.00	1
10	1.36	1	39	1.50	2			
11	1.30	1	40	2.12	2			
12	2.01	2	41	1-32	1			
13	1.14	1	42	1.41	2			
14	6.17	6	43	1.42	2			
15	2.14	2	44	1.08	1			
16	1.13	1	45	1.02	1			
17	2.30	2	46	1.30	1			
18	1.16	1	47	1.15	1			
19	8.08	8	48	1.20	1			
20	1.20	1	49	1.45	2			
21	1.14	1	50	1.15	1			
22	1.11	1	51	1.17	1			
23	2.19	2	52	2.01	2			
24	1.17	1	53	1.16	1			
25	1.16	1	54	1.45	2			
26	1.43	2	55	1.51	2			
27	1.41	2	56	1.06	1			
28	1.42	2	57	1.14	1			
29	1.29	1	58	1.20	1			

Let NA= Number of Arrivals, SRT= Service Time (in minute), AST=Approximated Service Time

Table C4. Service time and approximated service time for data collected on 23rd May, 2019

NA	SRT	AST	NA	SRT	AST	NA	SRT	AST
1	2.13	2	29	1.30	1	57	1.42	2
2	1.27	1	30	1.33	1	58	1.00	1
3	1.20	1	31	1.42	2	59	1.01	1
4	2.42	3	32	1.03	1	60	1.37	1
5	1.23	1	33	1.18	1	61	2.58	3
6	4.18	4	34	1.10	1	62	1.15	1
7	6.31	6	35	1.30	1	63	2.35	2
8	1.42	2	36	2.11	2	64	2.16	2
9	1.03	1	37	2.14	2	65	2.23	2
10	1.19	1	38	6.09	6			
11	2.46	3	39	1.19	1			
12	1.23	1	40	1.23	1			
13	1.34	1	41	1.37	1			
14	1.01	1	41	1.46	2			
15	2.07	2	43	1.32	1			
16	1.34	1	44	1.17	1			
17	1.17	1	45	2.00	2			
18	1.32	1	46	1.00	1			
19	1.21	1	47	1.42	2			
20	2.30	2	48	1.16	1			
21	1.34	1	49	3.04	3			
22	1.46	2	50	1.07	1			
23	1.02	1	51	1.41	2			
24	1.32	1	52	2.39	2			
25	2.08	2	53	1.36	1			
26	1.13	1	54	2.12	2			
27	2.32	2	55	1.45	2			
28	1.41	2	56	4.08	4			

Let NA= Number of Arrivals, SRT= Service Time (in minute), AST=Approximated Service Time

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