Electromagnetic Theory: Some Philosophical and Mathematical Problems of the Wave and Helmholtz Equations

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Abstract

In this article some intriguing aspects of electromagnetic theory and its relation to mathematics and reality are discussed, in particular those related to the suppositions needed to obtain the wave equations from Maxwell equations and from there Helmholtz equation. The following questions are discussed. How is that equations obtained with so many irreal or fictitious assumptions may provide a description that is in a high degree verifiable? Must everything that is possible to deduce from a theoretical mathematical model occur in the world? Does everything that takes place in the world have a mathematical description?

Keywords

Philosophy of Science, Philosophy of Physics, Electromagnetism

1. Introduction

Electromagnetic theory is a well-established fundamental part of physics. All calculations carried out with this theory are very precise and in exceptional agreement with observations. Besides, practically all technology mankind nowadays uses, from the most elementary to the most sophisticated, e.g. from electric vehicles to cellular phones and spacecrafts, is based on its use and validity. Nevertheless, it has, as other well accepted parts of physics such as quantum and statistical mechanics, some interesting philosophical uncertainties and basic difficulties.

As scientists, we take for granted that electromagnetic theory is valid anywhere in the universe. This is so because it is mostly accepted that this theory is a fundamental one, which means that it is spatiotemporally unrestricted, as has

been discussed by Klemke et al. (1998). Nevertheless, it is known that there are important objections to this fundamentalist view, such as for example, the nomological pluralism of Cartwright (2005, 1983), as has been discussed by Aboites (2022) and Rodríguez-Yañez et al. (2021) among others. On the other hand as expected, this theory is a clear example of a theory that makes scientific explanation distinctively mathematical, the need of this has been debated by Lange (2013) among others.

At any rate at least since Galilei, our best scientific theories are mathematically expressed. Even though some may think that the objects of mathematics are ideal or fictional entities, it is generally accepted that mathematics is indispensable for an account of the physical world. In fact, it is not at all obvious how we could express our scientific theories without using a mathematical vocabulary. Albeit some interesting attempts have been made in this direction, such as Field (1980) fictionalism, however this view is far from being generally accepted among scientists.

We are compelled to accept mathematical entities as part of our philosophical ontology. Putnam (1971) has shown that classical and modern physics require measurable quantities expressed as real numbers and the relations between these quantities are expressed through equations. From this he concludes that it is not possible to do science without real numbers, therefore real numbers exist as well as functions. This view will be further discussed in the next section.

As Shapiro (2000) and Hart (1998) among many others have shown, the mathematical description of the universe gives rise to many deep, interesting and difficult scientific and philosophical questions, such as: What is the mathematical description of a physical event? How is it that mathematical objects may be related to the physical world in such a way that the application of mathematics to the world is possible? Why mathematics is essential to science? How is that the mental constructions of mathematics allow us to clarify facts of the external material universe? How can a mathematical fact be used as an explanation to physical facts? Clearly, this is a fascinating and very broad discussion but far beyond our present aim.

In this article we will focus in discussing and exemplify mainly the following questions, which are viewed in the context of the mathematical description of electromagnetic theory. How it is that equations obtained with so many irreal or fictitious assumptions provide a description that is in high degree verifiable? Must everything that is possible to deduce from a theoretical mathematical model occur in the world? Does everything that takes place in the world have a mathematical explanation? Many of these questions have several answers or none. Our purpose is to provide a detailed example of these answers applied to the case of electromagnetic theory and some of its applications. Following the so-called indispensability argument, which will be detailed latter on, we assume as valid real analysis and the mathematical structure of calculus and differential equations, normally used to express electromagnetic theory.

In addition, in what follows we also take a scientific realist account of the world as discussed by Suppe (1989), Putnam (1990), Kitcher (1993), and Laudan (1996), among others. This is by far, among scientists, the most commonly accepted view of the world and the universe. In the next section the main arguments in favor and against scientific realism will be briefly reviewed. At this moment we may briefly advance that probably the most significant argument for scientific realism is the non-miracle argument set forward by Hilary Putnam (1975). This argument is based on the awareness that scientific realism is the only philosophical stance that does not makes of the success of science a miracle. As we know we have astonishing scientific achievements such as nuclear energy, space travel further away from our solar system, telecommunications and many others, therefore we may ask: How can we explain the incredible success of science? The answer provided by a scientific realist is that this is so because scientific theories are correct. If these theories where not correct i.e. if they would not truly describe the world, their success would be a miracle.

A summary of this article is therefore the following. In the next section a brief account of realism and some of its main philosophical alternatives in physics and in mathematics is provided. We believe that this is important in order to provide a short but straightforward justification, within a broad perspective, of the realist account of the world and also of the Quine-Putnam indispensability argument. Both of these hypotheses are accepted as valid in what follows in this article. We may add that based on our scientific professional experience it is recognized, forthrightly without proof, that these theses are far more widely accepted among scientists than their opposites. Further, in the following section, the physical and mathematical assumptions to go from Maxwell equations to wave equations, and from the wave equations to the Helmholtz equations, will be discussed. Finally, the conclusions are presented.

2. Realism in Physics and in Mathematics

As was previously mentioned, the scientific realist account of the universe has been discussed by Suppe (1989), Putnam (1990), Kitcher (1993), and Laudan (1996) and Leplin (1997) among others. In order to differentiate scientific realism from any other non-realist position we take the following three *metaphysical, semantic and epistemic* thesis set forward by Psillos (2000):

1) The metaphysical stance assert that the world has a definite and mind-independent natural-kind structure

2) The semantic stance takes scientific theories at face-value, seeing them as truth-conditioned descriptions of their intended domain, both observable and unobservable. Hence, they are capable of being true or false. Theoretical assertions are not reducible to claims about the behavior of observables, nor are they merely instrumental devices for establishing connections between observables. The theoretical terms featuring in theories have putative factual reference. So, if scientific theories are true, the unobservable entities

they posit populate the world.

3) The epistemic stance regards mature and predictively successful scientific theories as well-confirmed and approximately true of the world. So, the entities posited by them, or, at any rate, entities very similar to those posited, do inhabit the world.

(Psillos (1999), pp. *xix*)

The first thesis is obviously of metaphysical character because we cannot prove that the world has a mind-independent structure. This is something that the scientific realist decides to accept as a not proved fact. The second thesis guarantees that scientific realism is different from instrumentalism and other empirical reductionist descriptions. Finally, the third thesis distinguish scientific realism from other agnostic or sceptic forms of empiricism. This implies that science contains theoretical truths as well as observational truths. For example, as will be further discussed, electric and magnetic phenomena are described by Maxwell's electromagnetic theory which accepts theoretical entities such as charges, fields, currents and others. The scientific realist will take those theoretical entities as part of reality whereas the anti-realist will deny this. Following this line of argument Gutierrez-Canales et al. (2019) presented a case study about atomic theory from the perspective of scientific realism.

As we may see, scientific realism is a kind of realism stating that scientific theories truthfully, or approximately truthfully, describe observable and non-observable entities of the external world. Instrumentalism is an important anti-realist thesis. There, scientific theories are only predictive instruments useful to relate different observable events of the world and, of outmost importance, the theoretical elements of scientific theories do not describe reality. Therefore, as stated by Chalmers (1999), for an instrumentalist the molecules in movement of kinetic theory are only useful fictions as well as the charges and fields of electromagnetic theory. The most important criticism to instrumentalism is based on the distinction between observable and non-observable entities.

Doubtless Hilary Putnam (1975)'s non-miracle argument is the most significant argument in favor of scientific realism. According to this argument scientific realism is the only philosophical posture that does not make the success of science a miracle. In order to explain the achievements of science a scientific realist will argue that this is so because scientific theories are correct. If they would not truthfully describe the world, their accomplishments would be a miracle. For the scientific realist there only two possible ways to explain the success of science, either this is due to a miracle, a miracle of truly cosmic proportions, or simply, scientific theories are successful because these theories are mostly correct and describe the reality of the world.

An important objection against the non-miracle argumentation is the constructive empiricism of Van Fraassen (1980). In this view the success of science argued by the scientific realist is not relevant. He argues that scientific theories are analog to well adapted living organisms and since only well adapted organ-

isms survive it is natural and not surprising to have successful scientific theories, those who are not or were not successful do not exist and are not used anymore. Van Fraassen sets the objective of science only on empirical agreement. Unlike instrumentalism, in constructive empiricism theories are understood in the same way as in scientific realism, that is, sentences are taken to have standard truth-conditions, with terms purporting to refer to genuine entities and properties; but to accept the theories requires only accepting observable entities and what the theories imply about them. The constructive empiricist will hold an agnostic attitude about any non-observable phenomena. Therefore, constructive empiricists will take as true or false statements about non-observable entities but they do not need to accept that they exist. An oversimplified example is the following: A scientific realist will take the theory of Bohr atomic model as a truth description of reality accepting the existence of atoms made up of electrons orbiting a nucleus. The instrumentalist will take Bohr atomic model not as a description of reality but only as a predictive instrument useful to describe experimental observations such as the spectra emitted by excited atoms e.g. Lyman, Balmer, Paschen, Brackett and other spectroscopy series. On the other hand, the constructive empiricist will take Bohr model as a description of reality but will take an agnostic position about non observable components of the model such as electrons and energy quanta.

The most important argument against constructive empiricism was provided by Alan Musgrave (1985). It goes as follows: In order to be able to distinguish between observable and non-observable statements the constructive empiricist must accept propositions such as "X is non-observable", which is precisely a proposition about non-observable entities and therefore unacceptable for a constructive empiricist due to his agnosticism about non-observables. Therefore, since a distinction between observable and non-observable is fundamental to the constructive empiricist, and since this distinction is incompatible with what one may believe, Musgrave considers the constructive empiricism of Van Fraassen as untenable.

Another important argumentation against scientific realism is the pessimistic induction set forward by Laudan (1985). This argument is the result of the historic analysis of science. In any discipline, not only in physics, there is a constant replacement of old theories by new ones, this due to the constant development of scientific knowledge. Many theories of the past are now considered false even though some of them were able to predict observable phenomena. From induction it follows the conclusion that it is probable that our scientific theories will eventually be replaced by new theories in the future.

Realism in mathematics is the position stating that mathematical objects, objectively exist independent of the mind of mathematicians, see for example Aboites (2008a), Anglin (1994), George & Velleman (2002), Körner (1960), Shapiro (2000). By mathematical object realists mean objects that do not belong to the space-time of the external world, which are acausal, eternal and indestructible.

The number "5" or the set of natural numbers are examples of what a realist will consider a mathematical object which is acausal, eternal and indestructible.

An idealist will agree that mathematical objects exist but will hold that they depend on human minds. Therefore, an idealist will agree with the proposition "If there were not minds, there would not be mathematical objects", but a realist will deny it. The position call Platonism is realism in ontology. In the dialogue *Sophist*, Plato held that "among the things that exist we include number in general" and in the *Theaetetus,* "Yes, number must exist if anything does". An important question for a realist is: How could human mind have knowledge of acausal, eternal, indestructible objects which in addition are outside of space and time? Even more, how could those objects be related to the external world so we can provide a description of them? From this point of view the realm of mathematics is *a priori* and independent of human experience. This would not be accepted by an antirealist.

Realism about truth-value is a position where mathematical propositions have an objective truth-value independent of the minds, languages and conventions of mathematicians. On the other hand, antirealism about truth-value holds that if mathematical propositions have a truth-value this will depend on the mind of the mathematicians. The relation between realism in ontology and realism in truth-value is in detail discussed by Benacerraf (1973).

Kant (1787) held that mathematics is known independent of sense experience and therefore is *a priori*. Also, since mathematical truths can not be known from the analysis of concepts, they are synthetic. Two alternatives to Kantian position are that mathematics is empirical and therefore *a posteriori*, or that mathematics is analytic. The first point of view was held, for example, by Mill (1843), whereas the second belongs mainly to the logicism proposal of Frege (1879, 1884) and Russell (1903) where the purpose is to reduce mathematics to logic.

Another important view on mathematics is formalism, here it is held that the essence of mathematics lies in symbol manipulation. Therefore, about any branch of mathematics a mathematician only needs a list of symbols and the rules for their manipulation. This is everything that can be say about such branch of mathematics. From the formalist point of view mathematics is not and can not be about "something" further than symbols and its rules. This position is widely accepted by philosophers and mathematicians. Some radical forms of formalism hold that mathematical symbols have not sense, anymore that the pieces of a chess game board. Other less radical forms accept that mathematical symbols may have a sense but this is irrelevant to do mathematics. It has been pointed out by Shapiro (2000) that formalism solve, in fact avoids, some difficult metaphysical and epistemological problems. For example, the question: What is mathematics about? Has the answer: About nothing! The question: What are numbers, sets, etc.? Has the answer: They do not exist, or if they do, they could equally well not exist! The question: What is mathematical knowledge? Has the answer: It is the knowledge of a game based on symbols and rules, or the knowledge

of the results obtained in this game! An interesting question is: If mathematics is nothing else but a game, how is possible that this game is useful in science? From the formalist point of view the question about why mathematics is useful to describe the real world has no answer. A soft version of formalism is Hilbert (1899) deductivism, where he states that the practice of mathematics consists in obtaining the logical consequences of uninterpreted axioms. The basic idea is to ignore interpretations and to concentrate in inferences. Therefore, the question of how a branch of mathematics can be applied? Has the answer: By finding interpretations that make the axioms true. Famously, Hilbert said in a seminar:

> In a proper axiomatization of geometry, one must always be able to say instead of "points, straight lines, and planes", "tables, chairs and beer mugs"
>
> <div align="right">Hilbert (1935: pp. 403)</div>

In the contemporary scene there are two main positions in the philosophy of mathematics. Those who hold that numbers, sets and other mathematical objects exist independently of the mind, language and conventions of the mathematician, and those who do not. The members of the first group are realists, notable members are Plato (1994), Frege (1879, 1884), Gödel (1931, 1933), Quine (1981), Putnam (1967, 1971), Hale (1987), and Maddy (1990). Noteworthy members of the second group are Field (1980), Chihara (1990) and Burges & Rosen (1997).

Field (1980) claims that there is only one serious argument for the existence of mathematical entities; the indispensability argument of Quine-Putnam. Putnam (1971) believes that we are forced to accept mathematical entities as part of our philosophical ontology. He has highlighted that science require of measurable quantities expressed as real numbers and also that the relation between these quantities is stated through equations. His conclusion is that it is not possible to do science without real numbers, therefore real numbers exist as well as mathematical functions. This is the line of argument of the so-called Quine-Putnam "indispensability argument", its main premises are:

> 1) Real analysis refers to, and has variables that range over, abstract object called "real numbers". One who accepts the truth of the axioms of real analysis is committed to the existence of these abstract entities.
> 2) Real analysis is indispensable for physics. That is, modern physics can be neither formulated nor practiced without statements of real analysis.
> 3) If real analysis is indispensable for physics, then one who accept physics as true of material reality is thereby committed to the truth of real analysis.
> 4) Physics is true, or nearly true.
>
> <div align="right">(Shapiro, 2000: p.228)</div>

In what follows in this article we take a scientific realist position and also accept the indispensability argument of Quine-Putnam. This is by far the position taken by most scientists in the world. Almost any scientific article published, in the so-called "hard-sciences", takes for granted realism and the indispensability argument. Nevertheless, it should not be forgotten that, as convincing as these

arguments may be, they are only working hypothesis. However, it should also be stressed, that not to accept those hypotheses usually takes scientific research to a dead end.

The mathematical account of consciousness is an interesting and controversial example, among others, where some would question whether equations can describe this phenomenon. However many others, such as Aboites (2008b), see no problem on this. Some believe that there could be domains where the equation-ability of reality is doubtful. Certainly most physicalists would not agree.

3. Assumptions to Go from Maxwell Equations to Wave Equations

Lev Landau in his well know *Course of Theoretical Physics*, when dealing with electrodynamics, states that:

> Like all macroscopic theories, the theory of electromagnetic fields deals with physical quantities averaged over elements of volume which are "physically infinitesimal", ignoring the microscopic variations of the quantities which result from the molecular structure of matter.
>
> (Landau, 1981: 1)

The empirical macroscopic Maxwell equations of continuous media for the electric E and magnetic H fields are written as (Born & Wolf, 1980: p. 1):

$$\operatorname{div} E = \sigma/\varepsilon_o \tag{1-a}$$

$$\operatorname{div} H = 0 \tag{1-b}$$

$$\operatorname{curl} E = -\mu_o\, \partial H/\partial t \tag{1-c}$$

$$\operatorname{curl} H = \varepsilon_o\, \partial E/\partial t + J \tag{1-d}$$

where σ and J are the charge density and current of the media, and ε_o and μ_o the electric permittivity and magnetic permeability of vacuum. These equations represent in the given order:

Gauss Law, Equation (1-a), stating that any electric charge density will produce an electric field, i.e. the electric field E of the left hand side of the equation is produced by the charge density σ of the right hand side.

Nonexistence of magnetic monopoles Law, Equation (1-b), stating this fact. This means that magnetic fields do not require the existence of magnetic monopoles i.e. the magnetic field H of the left hand side of the equation has no sources on the right hand side.

Faraday Induction Law, Equation (1-c), which states that the change in time of a magnetic field will produce an electric field i.e. the electric field E of the left hand side of the equation is a consequence of the change in time of the magnetic field H of the right hand side.

Ampere Law, Equation (1-d), which states that a magnetic field will be produced either by change in time of an electric field or by an electric current. i.e. the magnetic field H of the left hand side of the equation is produced either by

the change in time of the electric field E of the right hand side or by an electric current J.

Electromagnetic waves are produced by electric charges in motion such as in a dipole antenna which is the simplest and most widely used class of antenna, but once they are produced they propagate. The propagation of electromagnetic waves is described by the wave equations. In order to obtain the electromagnetic wave equations from Maxwell equations Equations (1) several important assumptions are required. First, it is necessary to assume that there is zero charge density and zero currents ($\sigma = 0$, $J = 0$) therefore Equation (1-a) will take a similar form as that of Equation (1-b), and Equation (1-c) will take a similar form as that of Equation (1-d). Under these assumption Equations (1) take a very beautiful and symmetric form and can be written as:

$$\operatorname{div} E = 0 \tag{2-a}$$

$$\operatorname{div} H = 0 \tag{2-b}$$

$$\operatorname{curl} E = -\mu_{o}\, \partial H / \partial t \tag{2-c}$$

$$\operatorname{curl} H = \varepsilon_{o}\, \partial E / \partial t \tag{2-d}$$

We may ask how accurate it is to assume that there are zero electric charges and currents. As we know charge pair production is the creation of a subatomic particle and its antiparticle from a neutral boson. This is so because charge conservation requires the production of a particle and its antiparticle. In this way the energy of a photon can be converted into an electron–positron pair according to the process:

$$\gamma \rightarrow e^{-} + e^{+} \tag{3}$$

As we can see, in pair production, a photon creates an electron and a positron. In addition, in the pair production process, the photon disappears. In the above equation a positron has the same mass as an electron but has a positive charge. Since a photon has no mass, it is considered that pair production is an example of creating matter from pure energy.

However, this process also means that one may never be sure that vacuum is free form charges and therefore form currents (since currents are the result of moving charges). Therefore, the above assumptions of zero charges $\sigma = 0$ and not currents $J = 0$, needed to go from Equations (1) to Equations (2), are not guaranteed anywhere in space and at any time, in the universe. However, both conditions are required to go from Equations (1) to Equations (2)!

From Equations (2) it is straightforward to obtain the wave equations for the electric E and the magnetic field H. These two equations describe all electromagnetic known wave phenomena; they are:

$$\nabla^2 E = \left(1/c^2\right) \partial^2 E / \partial t^2 \tag{4-a}$$

$$\nabla^2 H = \left(1/c^2\right) \partial^2 H / \partial t^2 \tag{4-b}$$

where ∇^2 is the Laplacian. These equations state that, in the absence of charges

and currents, electric and magnetic fields oscillate and propagate in vacuum at the speed of light *c*.

4. Helmholtz Equation

The temporal and spatial electric $E(r,t)$ and magnetic $H(r,t)$ fields of Equation (4) may be assumed to be made up of separable independent functions in space and time, where the temporal part oscillates at a single frequency ω, as:

$$E(r,t) = R(r)\exp(i\omega t) \tag{5-a}$$

$$H(r,t) = R(r)\exp(i\omega t) \tag{5-b}$$

From a mathematical point of view, it is known that to ask for a monochromatic oscillation at a single frequency ω requires a Fourier transform with a delta of Dirac δ(ω) in frequency, which on time requires to extent along an infinite time, which clearly is physically impossible. Therefore, to assume monochromatic functions in frequency is only an idealization far from what is real or possible, one never has this in the real world.

It is also important to ask about the validity of assuming that the electric and magnetic fields may be expressed as the product of independent spatial *R*(*r*) and temporal exp(*iωt*) functions, as written above. By independence of variables, we mean that observing one variable does not tell us anything about the other. Is this the case? Mathematically, when two variables taken together, form a continuous random vector, independence can be verified by means of the following proposition:

> Two random variables *X* and *Y*, forming a continuous random vector, are independent if and only if: $f_{XY}(x,y) = f_X(x)f_Y(y)$, $\forall x, y \in \mathbb{R}$, where $f_{XY}(x,y)$ is their joint probability density function and $f_X(x)$ and $f_Y(y)$ are their marginal probability density functions
>
> (Taboga, 2021: p. 105)

With these two assumptions, where one is merely an approximation (monochromatic waves), and the other is just a mathematical simplifying working assumption (independent functions), the Helmholtz equation follows directly from equations (5):

$$\nabla^2 R(r) + k^2 R(r) = 0 \tag{7}$$

where *k* is the wave vector given by the dispersion relation $k^2 = \omega^2/c^2$. This equation is extremely important in science and technology and also has deep philosophical consequences on its own. Helmholtz equation provides the spatial part of any monochromatic electromagnetic wave, even though we know that in the real world there are not monochromatic waves but just, using lasers, approximations to this. In principle any function *R*(*r*) satisfying this equation is a possible spatial field distribution for an electromagnetic wave. Known solutions to this equation include: Plane waves $R_1(r)$. These waves represent infinite flat plane sheets traveling in space at the speed of light. Another well-known solution is:

Spherical waves $R_2(\pmb{r})$. These waves represent concentric uniform spherical waves expanding from an origin to infinite. The light provided by a small incandescent lamp is a good practical example. Another solution of Helmholtz equation is: Cylindrical waves $R_3(\pmb{r})$. These waves represent cylindrical electromagnetic fields radially expanding from an axis where they originate. These three examples are some of the most practical and common solutions to Equation (7), however the technological development of the laser in the middle of the 20th century by Maiman et al. (1961) required a new solution to Helmholtz equation able to describe a laser beam. This new solution, $R_4(\pmb{r})$, was found and is called a Hermite-Gauss beam, or a gaussian beam, it represents with great precision a laser beam. It is interesting to recall that in 1977 a science fiction film, *Star Wars*, challenged world scientist with the question: The "laser swords" shown in this film are possible in the real world? (This, from the point of view of their light spatial distribution and not from the one related to their fictitious destructive ability). To mathematically answer this question, it was necessary to look for the existence of new solutions to Helmholtz equation, call them $R_5(\pmb{r})$, with the spatial shape of a finite-size sword. These solutions exist indeed and were discovered by Durnin et al. (1987), they are call, non-diffractive Bessel beams.

Given the above real examples we may ask: Does any mathematical solution to the Helmholtz equation represent a spatial distribution for an electromagnetic wave that may exist in the real world? How could we know this? Whatever given answer, positive or negative, how could this be proved? What we know, till now, is that for any physically known spatial distribution of an electromagnetic wave we have a solution to Helmholtz equation.

As we can see, we need to distinguish between two different and important questions:

1) Is there a solution of the Helmholtz equation for any experimentally observed spatial distribution of an electromagnetic wave?
2) Given any mathematical solution of the Helmholtz equation, do we have a factual representation of it in the real world?

There is no way we could definitely and *a priori* answer any one of these questions. Most scientists will believe that the first question has a positive answer, but surely will not dare to believe the same concerning the second question. This is so because from the numerous mathematical solutions that Helmholtz equations may have, there is nothing *a priori* linking the abstract mathematical solution of this equation with our real world.

In science we use mathematical symbols and structures in order to elaborate precise representations of events occurring in the real world. Therefore, from the history of science we may argue that the first question will always have a positive answer. This is so because scientists will take whatever step needed in order to provide a mathematical model or representation of reality using, to achieve this goal, any present mathematical knowledge. Also, if necessary, scientists will develop whatever new mathematical tool to deal with the new problem. Since

many centuries this has been the standard scientific procedure and so we have many mathematical tools available used to describe the universe. This has been the case, for example of imaginary numbers, matrix algebra, complex variable, Lee algebras, Fourier analysis and many others, with applications in electromagnetic theory and circuits, quantum mechanics, relativity and cosmology among others.

We may not be able to explain and rationalize why a positive answer to the first question is possible, but we know for sure that it is possible, the history of science and technology leads us to provide a clear and positive answer to this question. Evidently this assumption is an argument based on induction which we may state as follows: Since we have been able to develop mathematical scientific theories for every observed fact of nature, we will be able to continue to do so.

To attempt to answer the second question is far more difficult since many mathematical equations and their solutions have sense in the mathematical realm but may, or may not, have any in the physical world. Our conjecture is that, *a priori*, this cannot be known.

5. Conclusion

In this article starting from a scientific realist view and accepting the indispensability argument of Quine Putnam, some of the most important fundamental assumptions of electromagnetic theory are summarized and discussed, in particular those required to go from Maxwell equations to wave equations and from wave equations to Helmholtz equations.

As we have seen, the electromagnetic theory is considered to be one of the greatest successes of scientific endeavor due to its ability to explain and predict within its realm what takes place in the world. Nevertheless, it may seem surprising how accurate electromagnetic theory is even though it contains some unrealistic assumptions such as zero charge and zero currents in order to obtain the wave equations. The explanation to this fact surely lies on the circumstance that the events non fulfilling these assumptions are rare, therefore the assumed vacuum conditions required to obtain the wave equations are mostly and in a high degree satisfied. This is also valid about the assumptions of monochromaticity and independence of spatial and temporal functions required to obtain Helmholtz equation. Even though these assumptions are not perfect they are enough to provide an extremely accurate, but approximate description of the world. This observation is in agreement with Leplin (1997) scientific realism remark stating that; "the (approximate) truth of a scientific theory is the only possible explanation of its predictive success".

Some questions related to the realm of the philosophy of mathematics and Helmholtz equation are also debated such as those relevant to the discussion of the relation between the state of affairs of the world and the mathematical theories that describe it. Two questions are posed. 1) Is there a solution of the

Helmholtz equation for any experimentally observed spatial distribution of an electromagnetic wave? And, 2) Given any mathematical solution of the Helmholtz equation, do we have a factual representation of it in the real world? From an inductive argument, a positive answer is poised to the first question. However, it is conjectured that for the second question an *a priori* answer cannot be provided.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

Aboites, G., Rodríguez-Yañez, A., & Aboites, V. (2022). Ceteris Paribus Laws and the Concept of Capacity in the Philosophy of Science of Nancy Cartwright. *Open Insight, 13,* 59-73. http://openinsight.com.mx/index.php/open/article/view/512

Aboites, V. (2008a). *Introducción a la filosofía de la matemática*. Libros a Cielo Abierto.

Aboites, V. (2008b). Multistable Chaotic Dynamical Systems and Philosophy. *International Journal of Bifurcation and Chaos, 18,* 1821-1824. https://doi.org/10.1142/S0218127408021397

Anglin, W. S. (1994). *Mathematics: A Concise History and Philosophy*. Springer Verlag. https://doi.org/10.1007/978-1-4612-0875-4

Benacerraf, P. (1973). Mathematical Truth. *Journal of Philosophy, 70,* 661-679. https://doi.org/10.2307/2025075

Born, M., & Wolf, E. (1980). *Principles of Optics*. Pergamon Press.

Burges, J. P., & Rosen, G. (1997). *A Subject with No Object: Strategies for Nomilaistic Interpretation of Mathematics*. Oxford University Press.

Cartwright, N. (1983). *How the Laws of Physics Lie.* Clarendon Press. https://doi.org/10.1093/0198247044.001.0001

Cartwright, N. (2005). *The Dappled World*. Cambridge University Press.

Chalmers, A. F. (1999). *What Is This Thing Called Science* (p. 233)? Open University Press.

Chihara, C. (1990). *Constructibility and Mathematical Existence*. Oxford University Press. https://doi.org/10.1093/0198239750.001.0001

Durnin, J., Miceli, Jr. J. J., & Eberly, J. H. (1987). Diffraction-Free Beams. *Physical Review Letters, 58,* 1499-1501. https://doi.org/10.1103/PhysRevLett.58.1499

Field, H. (1980). *Science without Numbers*. Princeton University Press.

Frege, G. (1879). *Begriffsschrift* (tr. Van Heijenoort, 1967). Harvard University Press.

Frege, G. (1884). *Die Grundlagen der Aritmetik* (tr. 1960). The Foundations of Arithmetic.

George, A., & Velleman, D. J. (2002). *Philosophies of Mathematics*. Blackwell.

Gödel, K. (1931). Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme. *Montatshefte für Mathematik und Physick, 38,* 173. https://doi.org/10.1007/BF01700692

Gödel, K. (1933). *On Undecidable Propositions of Formal Mathematical Systems*. Raven Press.

Gutierrez-Canales, C. F., Obregón, O., & Aboites, V. (2019). Teoría atómica y realismo científico. *Revista Mexicana de Física E, 65,* 84-100. https://doi.org/10.31349/RevMexFisE.65.84

Hale, B. (1987). *Abstract Objects*. Blackwell.

Hart, W. D. (1998). *The Philosophy of Mathematics*. Oxford Readings in Philosophy.

Hilbert, D. (1899). *Grundlagen der Geometrie* (tr. 1959). Foundations of Geometry.

Hilbert, D. (1935). *Gesammelte Abhandlunger, Dritter Band*. Springer.

Kant, E. (1787). *Kritik der reinen Vernunft*. Suhrkamp.

Kitcher, P. (1993). *The Advancement of Science.* OUP.

Klemke, E. D., Hollinger, R., & Wyss, D. (1998). *Philosophy of Science*. Prometheus Books.

Körner, A. S. (1960). *The Philosophy of Mathematics*. Dover.

Landau, L. D. (1981). *Course of Theoretical Physics* (Vol. 8). Pergamon.

Lange, M. (2013). What Makes a Scientific Explanation Distinctively Mathematical? *British Journal for the Philosophy of Science, 64,* 485-511. https://doi.org/10.1093/bjps/axs012

Laudan, L. (1985). *A Confutation of Convergence Realism*. University Chicago Press. https://doi.org/10.1525/9780520337442-012

Laudan, L. (1996). *Beyond Positivism and Relativism.* Westview Press.

Leplin, J. (1997). *A Novel Defense of Scientific Realism*. Oxford University Press.

Maddy, P. (1990). *Realism in Mathematics*. Oxford University Press.

Maiman, T. H., Hoskins, R. H., D'Haenens, I. J., Asawa, C. K., & Evtuhov, V. (1961). Stimulated Optical Emission in Fluorescent Solids II: Spectroscopy and Stimulated Emission in Ruby. *Physical Review, 123,* 1151-1157. https://doi.org/10.1103/PhysRev.123.1151

Mill, J. S. (1843). *A System of Logic: Ratiocinative and Inductive.* University Press of the Pacific.

Musgrave, A. (1985). *Realism versus Constructive Empiricism*. University of Chicago Press.

Plato (1994). *The Collected Dialogues.* Princeton University Press.

Psillos, S. (1999). *Scientific Realism*. Routledge.

Psillos, S. (2000). The Present State of the Scientific Realism Debate. *The British Journal for the Philosophy of Science, 51,* 705-728. https://doi.org/10.1093/bjps/51.4.705

Putnam, H. (1967). Mathematics without Foundations. *Journal of Philosophy, 64,* 5. https://doi.org/10.2307/2024603

Putnam, H. (1971). *Philosophy and Logic*. Harper Torchbooks.

Putnam, H. (1975). *Mathematics, Matter and Method*. Cambridge University Press.

Putnam, H. (1990). *Realism with a Human Face*. Harvard University Press.

Quine, W. V. O. (1981). *Theories and Things*. Harvard University Press.

Rodríguez-Yañez, A., Aboites, V., & Rionda, L. M. (2021). Nancy Cartwright, una introducción a su filosofía de la ciencia. *Revista Mexicana de Física E, 18,* 168-177. https://doi.org/10.31349/RevMexFisE.18.168

Russell, B. (1903). *The Principles of Mathematics*. George Allen.

Shapiro, S. (2000). *The Philosophy of Mathematics*. Oxford University Press.

https://doi.org/10.1093/0195139305.001.0001

Suppe, F. (1989). *The Semantic Conception of Theories and Scientific Realism.* University of Illinois Press.

Taboga, M. (2021). *Independent Random Variables*. Lectures on Probability Theory and Mathematical Statistics, Kindle Direct Publishing.
https://www.statlect.com/fundamentals-of-probability/independent-random-variables

Van Fraassen, B. C. (1980). *The Scientific Image*. Oxford University Press.
https://doi.org/10.1093/0198244274.001.0001