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Transient Solution of Machine Interference Problem with an Unreliable Server under Multiple Vacations Policy

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Abstract

Consider the machine interference problem with an unreliable server under multiple vacations. There are M similar machines that are subject to breakdowns with a single server who is responsible for repairing the failed machines. Each machine fails completely at random with rate λ. When a machine fails, it is immediately sent to the service centre where it is attended to in order of breakdowns with a state dependent service rate. State dependent service rate is a situation where the rate of service depends on the number of customers present in the system. The machines operate independently but are subject to breakdowns. The service time distributions of the failed machines are assumed to be exponentially distributed with state dependent service rate μ_n. Where n is the number of failed machines. The Chapman-Kolmogorov differential equations obtained for the multiple vacations model is solved through ODE45 (Runge-Kutta algorithm of order 4 and 5) in MATLAB programming language. The transient probabilities obtained are used to compute the operational measures of performance for the systems. In the multiple vacations model the server will continue to take vacations until there is one failed machine in the system. The effects of $\lambda, \mu, \alpha, \beta$ and θ on the machine availability under different values of t for the multiple vacations is investigate; it is observe that the machine availability decreases with increase in time t. The CPU time for obtaining the transient results for the systems and the variance of the systems are reported in this work.

Keywords: Transient solution, machine interference problem, multiple vacations, ODE45 in MATLAB.

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1 Introduction

This paper discussed the machine interference problem with unreliable server under multiple vacations. In the machine interference problem there are M similar machines that are subject to breakdowns or fail with multiple vacations policy. Machines used in production process are subject to fail or breakdown, when the machine fails or breakdown it is sent to a repairer (server) to be repaired. This is done to meet up production and reduce loss of production in the system. In the machine interference problem the repairer (server) repair broken down machine and make it operational. If there are one or more broken down machines and the repairer is busy when another broken down machine needs service we say the machine interfere with each other's service.

In multiple vacations, if there is no failed machine waiting for service after a vacation, the server immediately leaves for another vacation. This pattern continues until he returns from vacation to find at least one failed machine waiting in queue for service.

Many authors have study the machine interference problem with unreliable server that goes on vacations ([1,3,6,9,12]). Most authors provide only steady result for the measures of performance for the system. But [5] gave an excellent survey of machine interference problem. However, the use of computer software to aid analysis of mathematical models developed for machine repair systems were not reported by [5]. This include the use of MAPLE by [10,11] and [12] who used MATLAB SOFTWARE. Also [5] suggested the need to develop measures of variation for the operational measures of performance.

Further [8] consider transient solution of machine interference problem with an unreliable server under single vacation policy. They model is formulated in terms of probability of finding n (where $0 \leq n \leq M$) failed machines in the system at time t ([4,5,6,7,8,13]).

We observe that most works on machine interference problem provide steady state for their system. From literature there are very few works dealing with transient state solution for the machine interference problem with server vacation. This motivates us to examine the transient solution of machine interference problem with unreliable server under multiple vacations.

The purpose of this paper is first to produce transient probability for the machine interference problem with unreliable server under multiple vacations. The transient probabilities obtain are used to find various operational measures of performance for the system. The second is to compute the Central Processing Unit time for obtaining the transient solution for the multiple vacations policy. The third is that, apart from finding the expected number of failed and operating machine we also obtain the variance and standard deviation of the number of failed and operating machines in the system which previous authors did not obtain.

The following time dependent operational measures of performance for the system are obtained: expected number of failed machines, expected number of operating machines, machine availability, expected idle period, expected busy period, operational utilization of the machine, also obtained are the variance of the expected number of failed machine and variance of expected number of operating machine in the system. The expected idle period in multiple vacations is zero. This is so because the server is not idle in multiple vacations.

2 Mathematical Formulations

We shall follow the treatment given by [8]. The transient state measures of performance for the machine interference problem with an unreliable server under single vacation policy are obtained by [8]. The system is a machine interference system in which the server is not completely available. The server can be broken down or can be on vacation. In the system there are M independent and identical machines which are subject to breakdown. Each machine breaks down according to a Poisson distribution with rate λ. When a machine fails, it is immediately sent to the service centre where it is attended to (service) in order of breakdowns. A broken down machine waits in the queue until it is repaired. The service times (repair times) distribution of the failed machines is assumed to be exponentially distributed with state dependent service rate μ_n, where n is the number of failed machines. The break down rate of the server is α while the repair rate is β. The breakdown process is Poisson. When the server is active, it is subject to breakdown with Poisson distribution rate α. When the server breaks down; it is immediately repaired. The repair times of the server is exponentially distributed with rate β.

When there are no failed machines queueing up for repair in the system, the server leaves the service point for a vacation of random length. The vacation length is exponentially distributed with parameter θ.

To obtain the differential equations for the system [6] represented the state of the system at epoch (t) by two variables namely: the condition of the server and the number of failed machines in the system. The server can be on vacation, or available to repair failed machines or broken down. Therefore, let the state of the system at epoch (t) be denoted by(i, n); $i = 0, 1, 2$; $0 \leq n \leq M$; where i is the condition of the server, and n is the number of failed machine in the system. When the server is on vacation $i = 0$; when the server is available $i = 1$, and when the server breaks down $i = 2$. Let.

$P_{0,n}$(t) The probability that there are n failed machine in the system when the server is on vacation at time t

$P_{1,n}$(t) The probability that there are n failed machine in the system when the server is active at time t

$P_{2,n}$(t) The probability that there are n failed machine in the system when the server breakdown at time t

With these notations, the differential equations for the machine interference problem with unreliable server under multiple vacations are presented:

$$P_{0,0}^{'}(t) = -(M\lambda)P_{0,0}(t) + \mu_1 P_{1,1}(t) \quad \ldots\ldots\ldots\ldots (1)$$

$$P_{0,n}^{'}(t) = -(\theta + (M-n)\lambda)P_{0,n}(t) + (M-n+1)\lambda P_{0,n-1}(t) \ldots\ldots\ldots\ldots (2)$$

$$1 \leq n \leq M-1$$

$$P_{0,M}^{'}(t) = -\theta P_{0,M}(t) + \lambda P_{0,M-1}(t) \ldots\ldots\ldots\ldots (3)$$

$$P_{1,1}{}'(t) = P_{1,1}(t)\big[-[(M-1)\lambda + \mu_1 + \alpha]\big] + \quad P_{1,2}(t)\mu_2 + P_{2,1}(t)\beta + P_{0,1}(t)\theta \ldots\ldots\ldots (4)$$

$$P_{1,n}^{'}(t) = P_{1,n}(t)\big[-[(M-n)\lambda + \mu_n + \alpha]\big] + P_{1,n-1}(t)[M-n+1]\lambda + P_{1,n+1}(t)\mu_{n+1} + P_{2,n}(t)\beta + P_{0,n}(t)\theta \;\; 2 \leq n \leq M-1 \quad \ldots\ldots\ldots\ldots (5)$$

$$P_{1,M}^{'}(t) = P_{1,M}(t)[-(\mu_M + \alpha)] + P_{1,M-1}(t)\lambda + P_{2,M}(t)\beta + \quad P_{0,M}(t)\theta \ldots\ldots\ldots\ldots (6)$$

$$P_{2,1}^{'}(t) = P_{2,1}(t)\big[-[(M-1)\lambda + \beta]\big] + \alpha\, P_{1,1}(t) \quad \ldots\ldots\ldots\ldots (7)$$

$$P'_{2,n}(t) = P_{2,n}(t)\big[-[(M-n)\lambda+\beta]\big] + P_{2,n-1}(t)[M-n+1]\lambda + P_{1,n}(t)\alpha$$
$$for\ 2 \le n \le M-1 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (8)$$

$$P'_{2,M}(t) = P_{2,M}(t)[-\beta] + P_{2,M-1}(t)\lambda + P_{1,M}(t)\alpha \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (9)$$

Where $\mu_n = 1 + \frac{n}{10}$

Note that the number of equations to be solved for the multiple vacations policy is 1+3M.

3 Transient Solution for the Multiple Vacations Policy

The transient probabilities $P_{i,n}(t); i = 0, 1, 2\ and\ 0 \le n \le M$ for the multiple vacations models can be obtained by solving the set of transient state difference –differential equations (1) to (9) above. We use MATLAB program for the solution of the difference –differential equations above. The transient probabilities are used to obtain the following measures of performance for the system:

1. The expected number of failed machine in the system at time (t) is

$$E[F(t)] = \sum_{n=0}^{M} nP_{0,n}(t) + \sum_{n=0}^{M} nP_{1,n}(t) + \sum_{n=1}^{M} nP_{2,n}(t)$$

2. Machine Availability -The machine availability is defined as the ratio of the average number of machines running to the total number of machines. The machine availability at time (t) ($M.A.(t)$) is given by the expression below

$$M.A.(t) = 1 - \frac{E[F(t)]}{M}$$

3. The expected number of operating machine at time (t) is

$$E[O(t)] = M - E[F(t)]$$

4. The expected idle period at time (t) is

$$E[I(t)] = P_{1,0}(t)$$

5. The expected number of vacation the server has at time (t) is

$$E[V(t)] = \sum_{n=0}^{M} nP_{0,n}(t)$$

6. The expected number of broken servers (when the servers are unreliable) at time (t) is

$$E[D(t)] = \sum_{n=1}^{M} P_{2,n}(t)$$

7. The expected number of busy period at time (t) is given by the expression

$$E[B(t)] = 1 - E[D(t)] - E[I(t)] - E[v(t)\}$$

8. Operative utilization (O. U.) is given by the expected number of busy period

$$O.U. = E[B(t)]$$

9. Variance: The variance of the number of broken down machine and the number of operating machines at time (t) are calculated using the expression

$$\sigma^2(t) = \sum_{n=0}^{M} n^2 P_{0,n}(t) + \sum_{n=0}^{M} n^2 P_{1,n}(t) + \sum_{n=1}^{M} n^2 P_{2,n}(t) - [E(F(t))]^2$$

Transient results for these measures of performance were obtained and several numerical experiments were performed.

3.1 Numerical Results for some M

The differential difference equations (1)-(9) representing the machine interference problems for the multiple vacations is readily solve using the ODE45 (Runge-Kutta algorithm of order 4 and 5) in MATLAB programming language.

The transient probabilities $P_{(i,n)}(t); i = 0,1,2 \; and \; 0 \leq n \leq M$ for the system are computed for each time t. The system starts empty with $P_{(0,0)}(0) = 1$ and $P_{(i,n)}(0) = 0 \; for \; all \; i = 0,1,2 \; and \; n = 0,1,2,\ldots,M$ as initial conditions [2].

Tables 1-4 show the different numerical results for the operational measures of performance for different values of time t for the multiple vacations policy. We observed that after some time t, the successive values of the operational measures of performance no longer varies, this means that the transient results were close to the steady state results. The results are presented in Tables 1-4 for different values of M**,** λ, μ, θ, α and β for the multiple vacations.

Table 1. Some performance measures for different values of t, M, λ, μ, θ, α and β for the multiple vacations

t	E(O)	E(F)	E(V)	E(I)	E(D)	M.A.	O.U.
			M=10, λ=0.15, μ=1.1, θ=1, α=0.05, β=10				
0	10.0000	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000
1	8.5025	1.4975	0.5926	0.0000	0.0024	0.8502	0.4050
2	7.7482	2.2518	0.3625	0.0000	0.0037	0.7748	0.6338
3	7.3772	2.6228	0.2648	0.0000	0.0042	0.7377	0.7310
4	7.1927	2.8073	0.2195	0.0000	0.0044	0.7193	0.7761
5	7.1011	2.8989	0.1976	0.0000	0.0045	0.7101	0.7979
6	7.0556	2.9444	0.1867	0.0000	0.0045	0.7056	0.8087
7	7.0330	2.9670	0.1812	0.0000	0.0046	0.7033	0.8143
8	7.0218	2.9782	0.1783	0.0000	0.0046	0.7022	0.8172
9	7.0164	2.9836	0.1768	0.0000	0.0046	0.7016	0.8187
10	7.0138	2.9862	0.1760	0.0000	0.0046	0.7014	0.8194
11	7.0127	2.9873	0.1756	0.0000	0.0046	0.7013	0.8199
12	7.0122	2.9878	0.1754	0.0000	0.0046	0.7012	0.8201
13	7.0120	2.9880	0.1753	0.0000	0.0046	0.7012	0.8202
14	7.0120	2.9880	0.1752	0.0000	0.0046	0.7012	0.8202
15	7.0120	2.9880	0.1752	0.0000	0.0046	0.7012	0.8202
16	7.0120	2.9880	0.1752	0.0000	0.0046	0.7012	0.8202
17	7.0121	2.9879	0.1752	0.0000	0.0046	0.7012	0.8202
18	7.0121	2.9879	0.1752	0.0000	0.0046	0.7012	0.8202
19	7.0121	2.9879	0.1752	0.0000	0.0046	0.7012	0.8202
20	7.0121	2.9879	0.1752	0.0000	0.0046	0.7012	0.8202

The expected idle period E(I) in the multiple vacations is zero. This is so because the server is not idle in multiple vacations policy.

Table 2. Some performance measures for different values of t, M, λ, μ, θ, α and β for the multiple vacations

t	E(O)	E(F)	E(V)	E(I)	E(D)	M.A.	O.U.
	M=9, λ=0.2, μ=1.1, θ=1, α=0.05, β=10						
0	9.0000	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000
1	7.2853	1.7147	0.5599	0.0000	0.0025	0.8095	0.4376
2	6.4427	2.5573	0.3239	0.0000	0.0038	0.7159	0.6723
3	6.0352	2.9648	0.2223	0.0000	0.0043	0.6706	0.7734
4	5.8423	3.1577	0.1754	0.0000	0.0045	0.6491	0.8201
5	5.7532	3.2468	0.1531	0.0000	0.0046	0.6392	0.8423
6	5.7127	3.2873	0.1424	0.0000	0.0046	0.6347	0.8530
7	5.6946	3.3054	0.1371	0.0000	0.0046	0.6327	0.8583
8	5.6866	3.3134	0.1344	0.0000	0.0047	0.6318	0.8609
9	5.6833	3.3167	0.1331	0.0000	0.0047	0.6315	0.8622
10	5.6820	3.3180	0.1325	0.0000	0.0047	0.6313	0.8628
11	5.6817	3.3183	0.1322	0.0000	0.0047	0.6313	0.8631
12	5.6817	3.3183	0.1321	0.0000	0.0047	0.6313	0.8633
13	5.6818	3.3182	0.1320	0.0000	0.0047	0.6313	0.8633
14	5.6819	3.3181	0.1320	0.0000	0.0047	0.6313	0.8633
15	5.6820	3.3180	0.1320	0.0000	0.0047	0.6313	0.8633
16	5.6821	3.3179	0.1320	0.0000	0.0047	0.6313	0.8633
17	5.6821	3.3179	0.1320	0.0000	0.0047	0.6313	0.8633
18	5.6822	3.3178	0.1320	0.0000	0.0047	0.6314	0.8633
19	5.6822	3.3178	0.1320	0.0000	0.0047	0.6314	0.8633
20	5.6822	3.3178	0.1320	0.0000	0.0047	0.6314	0.8633

Table 3. Some performance measures for different values of t, M, λ, μ, θ, α and β for the multiple vacations

t	E(O)	E(F)	E(V)	E(I)	E(D)	M.A.	O.U.
	M=8, λ=0.35, μ=1.1, θ=1, α=0.05, β=10						
0	8.0000	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000
1	5.6169	2.3831	0.4878	0.0000	0.0027	0.7021	0.5095
2	4.5269	3.4731	0.2400	0.0000	0.0039	0.5659	0.7561
3	4.0682	3.9318	0.1336	0.0000	0.0043	0.5085	0.8621
4	3.8921	4.1079	0.0871	0.0000	0.0044	0.4865	0.9085
5	3.8264	4.1736	0.0666	0.0000	0.0045	0.4783	0.9289
6	3.8018	4.1982	0.0575	0.0000	0.0045	0.4752	0.9380
7	3.7927	4.2073	0.0534	0.0000	0.0046	0.4741	0.9420
8	3.7895	4.2105	0.0516	0.0000	0.0046	0.4737	0.9438
9	3.7887	4.2113	0.0508	0.0000	0.0046	0.4736	0.9446
10	3.7888	4.2112	0.0505	0.0000	0.0046	0.4736	0.9450
11	3.7890	4.2110	0.0503	0.0000	0.0046	0.4736	0.9451
12	3.7892	4.2108	0.0503	0.0000	0.0046	0.4737	0.9452
13	3.7894	4.2106	0.0503	0.0000	0.0046	0.4737	0.9452
14	3.7896	4.2104	0.0503	0.0000	0.0046	0.4737	0.9452
15	3.7896	4.2104	0.0503	0.0000	0.0046	0.4737	0.9452
16	3.7897	4.2103	0.0503	0.0000	0.0046	0.4737	0.9452
17	3.7897	4.2103	0.0503	0.0000	0.0046	0.4737	0.9452
18	3.7897	4.2103	0.0503	0.0000	0.0046	0.4737	0.9452
19	3.7898	4.2102	0.0503	0.0000	0.0046	0.4737	0.9452
20	3.7898	4.2102	0.0503	0.0000	0.0046	0.4737	0.9452

Table 4. Some performance measures for different values of t, M, λ, μ, θ, α and β for the multiple vacations

T	E(O)	E(F)	E(V)	E(I)	E(D)	M.A.	O.U.
			M=6, λ =0.3, μ=1.1, θ =5, α =0.05, β =10				
0	6.0000	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000
1	4.3704	1.6296	0.2655	0.0000	0.0045	0.7284	0.7300
2	3.9702	2.0298	0.1678	0.0000	0.0049	0.6617	0.8273
3	3.8021	2.1979	0.1436	0.0000	0.0049	0.6337	0.8515
4	3.7200	2.2800	0.1338	0.0000	0.0049	0.6200	0.8614
5	3.6782	2.3218	0.1291	0.0000	0.0048	0.6130	0.8660
6	3.6565	2.3435	0.1268	0.0000	0.0048	0.6094	0.8684
7	3.6452	2.3548	0.1256	0.0000	0.0048	0.6075	0.8696
8	3.6393	2.3607	0.1250	0.0000	0.0048	0.6066	0.8702
9	3.6362	2.3638	0.1246	0.0000	0.0048	0.6060	0.8705
10	3.6346	2.3654	0.1245	0.0000	0.0048	0.6058	0.8707
11	3.6337	2.3663	0.1244	0.0000	0.0048	0.6056	0.8708
12	3.6333	2.3667	0.1243	0.0000	0.0048	0.6055	0.8708
13	3.6330	2.3670	0.1243	0.0000	0.0048	0.6055	0.8709
14	3.6329	2.3671	0.1243	0.0000	0.0048	0.6055	0.8709
15	3.6328	2.3672	0.1243	0.0000	0.0048	0.6055	0.8709
16	3.6328	2.3672	0.1243	0.0000	0.0048	0.6055	0.8709
17	3.6328	2.3672	0.1243	0.0000	0.0048	0.6055	0.8709
18	3.6328	2.3672	0.1243	0.0000	0.0048	0.6055	0.8709
19	3.6328	2.3672	0.1243	0.0000	0.0048	0.6055	0.8709
20	3.6328	2.3672	0.1243	0.0000	0.0048	0.6055	0.8709

After some time t=20, the successive values of the operational measures of performance for the system no longer vary. This means that the transient results were close to the steady state results. We compared such results with those of [6] in Table 5 for the multiple vacation policy.

Table 5. Comparing system characteristics of [6] results with our transient results for the multiple vacations

α=0.05 β=10	Ke (2006) Results	Time dependent results	Ke(2006) Results	Time dependent results	Ke(2006) results	Time depen-dent results	Ke(2006) Results	Time depen-dent results
(λ,θ)	(0.15,1.0)	(0.15,1.0)	(0.2,1.0)	(0.2,1.0)	(0.35,1.0)	(0.35,1.0)	(0.3,5.0)	(0.3,5.0)
M	10	10	9	9	8	8	6	6
E(F)	2.8974	2.9879	3.2358	3.3178	4.1560	4.2102	2.3678	2.3672
E(o)	7.1026	7.0121	5.7642	5.6822	3.8440	3.7898	3.6322	3.6328
E(v)	0.1937	0.1752	0.1434	0.1320	0.0525	0.0503	0.1370	0.1243
E(I)	0.0000	0.0000	0.0000	0.0000	0.0000	00000	0.0000	0.0000
E(D)	0.0040	0.0046	0.0043	0.0047	0.0047	0.0046	0.0043	0.0048
M.A.	0.7103	0.7012	0.6405	0.6314	0.4805	0.4737	0.6054	0.6055
O.U.	0.8023	0.8202	0.8523	0.8633	0.9427	0.9452	0.8587	0.8709
var		0.5096		0.6208		0.9613		0.2808
SD		0.7139		0.7879		0.9805		0.5299

SD means the standard deviation of the number of broken down machine.

Tables 6-10 below shows the effects of $\lambda, \mu, \alpha, \beta$ *and* θ on the machine availability under different values of t for the multiple vacation policy, we found that the machine availability decreases with increase in time t. This is to be expected. Initially all the machines are in good working condition. As elapsed time increases and due to wear and tear, some of the machines will break down. Hence the machine availability decreases with time. We also found out that from time t=10 to t=20 the machine availability no longer varies, it therefore means that steady state results are obtained through the transient approach.

Table 6. Effect of λ on the machine availability under different values of t for multiple vacations. $\mu = 1.0, \alpha = 0.05, \beta = 10, \theta = 1.0, M = 10$

	λ		
t	**λ =0.15**	**λ =0.2**	**λ =0.3**
	M.A.	**M.A.**	**M.A.**
0	1.0000	1.0000	1.0000
1	0.8496	0.8109	0.7384
2	0.7720	0.7128	0.6047
3	0.7328	0.6620	0.5393
4	0.7128	0.6361	0.5086
5	0.7026	0.6229	0.4940
6	0.6974	0.6161	0.4866
7	0.6948	0.6126	0.4826
8	0.6935	0.6106	0.4804
9	0.6928	0.6096	0.4791
10	0.6925	0.6090	0.4783
11	0.6924	0.6087	0.4779
12	0.6923	0.6085	0.4776
13	0.6923	0.6084	0.4774
14	0.6923	0.6083	0.4773
15	0.6923	0.6083	0.4773
16	0.6923	0.6083	0.4772
17	0.6923	0.6082	0.4772
18	0.6923	0.6082	0.4772
19	0.6923	0.6082	0.4772
20	0.6923	0.6082	0.4772
Var	0.5467	0.8899	1.5557

Table 7. Effect of μ on the machine availability under different values of t for multiple vacations. $\lambda = 0.15, \alpha = 0.05, \beta = 10, \theta = 1.0, M = 10$

	μ		
t	**μ =1.0**	**μ=2.0**	**μ =3.0**
	M.A.	**M.A.**	**M.A.**
0	1.0000	1.0000	1.0000
1	0.8496	0.8560	0.8618
2	0.7720	0.7968	0.8158
3	0.7328	0.7739	0.8018
4	0.7128	0.7649	0.7974
5	0.7026	0.7613	0.7960
6	0.6974	0.7599	0.7955
7	0.6948	0.7593	0.7954
8	0.6935	0.7591	0.7954
9	0.6928	0.7590	0.7954
10	0.6925	0.7590	0.7954
11	0.6924	0.7590	0.7954
12	0.6923	0.7590	0.7954
13	0.6923	0.7590	0.7954
14	0.6923	0.7590	0.7954
15	0.6923	0.7590	0.7954
16	0.6923	0.7590	0.7954
17	0.6923	0.7590	0.7954
18	0.6923	0.7590	0.7954
19	0.6923	0.7590	0.7954
20	0.6923	0.7590	0.7954
Var	0.5467	0.3077	0.2122

We observe that as the failure rate λ of operating machines increases the variance also increases and the machine availability decreases for the multiple vacations policy. That is the failure rate of the machines affects the variance and the machine availability in the system.

We observe that as the service rate μ of the machines increases the variance decreases and machine availability increases for the multiple vacations policy. The service rate also affects the variance and the machine availability in the system.

Table 8. Effect of α on the machine availability under different values of t for multiple vacation. $\lambda = 0.15, \mu = 1.0, \beta = 10, \theta = 1.0, M = 10$

t	α =0.01	α =0.05	α =0.1
	M.A.	M.A.	M.A.
0	1.0000	1.0000	1.0000
1	0.8502	0.8496	0.8488
2	0.7738	0.7720	0.7698
3	0.7358	0.7328	0.7290
4	0.7169	0.7128	0.7075
5	0.7077	0.7026	0.6962
6	0.7032	0.6974	0.6901
7	0.7012	0.6948	0.6868
8	0.7002	0.6935	0.6849
9	0.6999	0.6928	0.6839
10	0.6997	0.6925	0.6833
11	0.6997	0.6924	0.6830
12	0.6997	0.6923	0.6828
13	0.6998	0.6923	0.6827
14	0.6998	0.6923	0.6827
15	0.6998	0.6923	0.6826
16	0.6998	0.6923	0.6826
17	0.6998	0.6923	0.6826
18	0.6998	0.6923	0.6826
19	0.6998	0.6923	0.6826
20	0.6999	0.6923	0.6826
Var	0.5163	0.5467	0.5872

Table 9. Effect of β on the machine availability under different values of t for multiple vacation policy. $\lambda = 0.15, \mu = 1.0, \alpha = 0.05, \theta = 1.0, M = 10$

t	β =3.0	β =6.0	β =10.0
	M.A.	M.A.	M.A.
0	1.0000	1.0000	1.0000
1	0.8495	0.8495	0.8496
2	0.7714	0.7718	0.7720
3	0.7315	0.7324	0.7328
4	0.7109	0.7122	0.7128
5	0.7002	0.7019	0.7026
6	0.6947	0.6966	0.6974
7	0.6918	0.6940	0.6948
8	0.6902	0.6926	0.6935
9	0.6894	0.6919	0.6928
10	0.6890	0.6915	0.6925
11	0.6888	0.6914	0.6924
12	0.6887	0.6913	0.6923
13	0.6886	0.6913	0.6923
14	0.6886	0.6913	0.6923
15	0.6886	0.6913	0.6923
16	0.6886	0.6913	0.6923
17	0.6886	0.6913	0.6923
18	0.6886	0.6913	0.6923
19	0.6886	0.6913	0.6923
20	0.6886	0.6913	0.6923
Var	0.5623	0.5510	0.5467

We observe that as the break down rate α of the server increases the variance also increases and machine availability decreases for the multiple vacations policy. The break down rate also affects the machine availability and the variance in the system.

We also observe that as the repair rate β of the server increases the machine availability increases the variance decreases for the multiple vacation policy. The repair rate also affects the machine availability and the variance in the system.

We observe that as the vacation length θ of the server increases the machine availability increases the variance decreases. The vacation length also affects the machine availability and the variance in the system.

Table 10. Effect of θ on the machine availability under different values of t for multiple vacation. $\lambda = 0.15, \mu = 1.0, \alpha = 0.05, \beta = 10.0, M = 10$

t	θ =1.0	θ =2.0	θ =3.0
	M.A.	M.A.	M.A.
0	1.0000	1.0000	1.0000
1	0.8496	0.8467	0.8460
2	0.7720	0.7858	0.7925
3	0.7328	0.7571	0.7665
4	0.7128	0.7411	0.7511
5	0.7026	0.7313	0.7414
6	0.6974	0.7252	0.7352
7	0.6948	0.7212	0.7311
8	0.6935	0.7185	0.7284
9	0.6928	0.7168	0.7266
10	0.6925	0.7157	0.7254
11	0.6924	0.7149	0.7245
12	0.6923	0.7144	0.7240
13	0.6923	0.7141	0.7236
14	0.6923	0.7139	0.7234
15	0.6923	0.7137	0.7232
16	0.6923	0.7136	0.7231
17	0.6923	0.7136	0.7231
18	0.6923	0.7135	0.7230
19	0.6923	0.7135	0.7230
20	0.6923	0.7135	0.7229
Var	0.5467	0.4513	0.4156

In all we found that $\lambda, \mu, \alpha, \beta\ and\ \theta$ affects the machine availability and variance for the multiple vacations policy.

3.2 Discussions

Table 11 compared transient results for single vacation [8] and the multiple vacations policy for the machine interference problems with unreliable serve discuss here. We found that with the same parameters there is a slight different in the expected number of operating machine E(O), the expected number of failed machine E(F), expected number of vacation the server has E[V], expected idle period E[I], the expected number of broken server, the machine availability, operative utilization and the variance for the multiple vacation policy and that of the single vacation policy with time t. This is because the servers have to undergo several vacations in the multiple vacation policy. The expected idle period in multiple vacations is zero. This is so because the server is not idle in multiple vacations. The server will continue to take vacations until there is one failed machine in the system. While in the single vacation once the server comes back from vacation he waits idly until there is one failed machine in the system.

The figures below correspond to some of the results presented in Tables 1-10 above.

Figs. 1 and 2 correspond to the result in Table 7. Figs. 1- 2 show the effect of service rate on the expected number of operating and failed machine in the system. We found that from time 0 to 2 the expected number of failed and operating machines are the same with increase in service rate, but as time increases from time 2 to 20 the expected number of operating machines increase with increase in service rate (Fig. 1). In a similar manner with decrease in service rate the expected number of failed machines increases (Fig. 2).

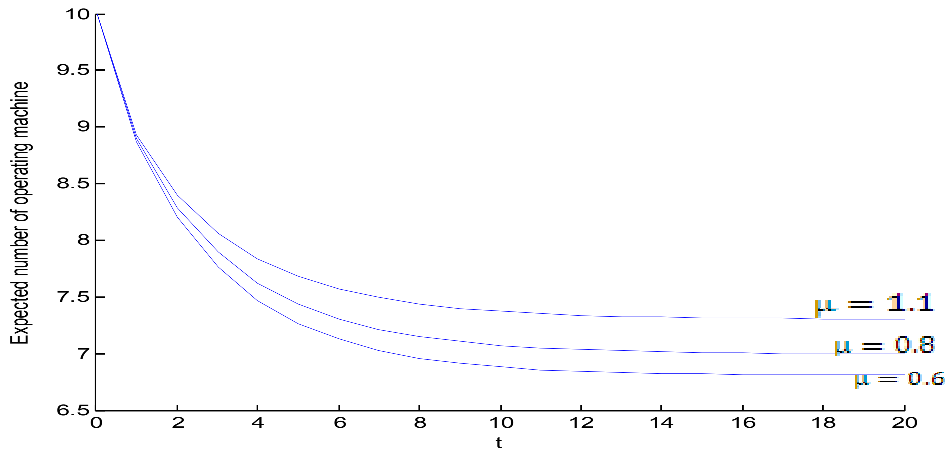

Fig. 1. The effect of service rate of failed machines on the expected number of operating machines in the system at time t when $\lambda = 0.15, \theta = 1, \alpha = 0.05, \beta = 10,\ M = 10$.

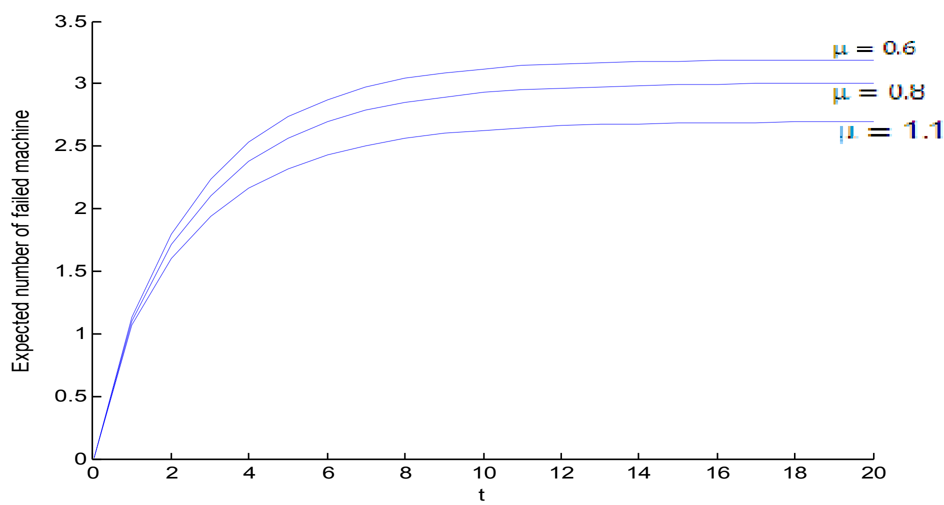

Fig. 2. The effect of service rate of failed machines on the expected number of failed machines in the system at time t when $\lambda = 0.15,\ \theta = 1, \alpha = 0.05, \beta = 10,\ M = 10$.

Figs. 3 and 4 correspond to the result in Table 6. Figs. 3-4 below shows the effect of failure rate on the expected number of failed and operating machine in the system. We found that with increase in failure rate the expected number of failed machines in the system also increases with time (Fig.

3). While the expected number of operating machine decreases with increase in failure rate (Fig. 4).

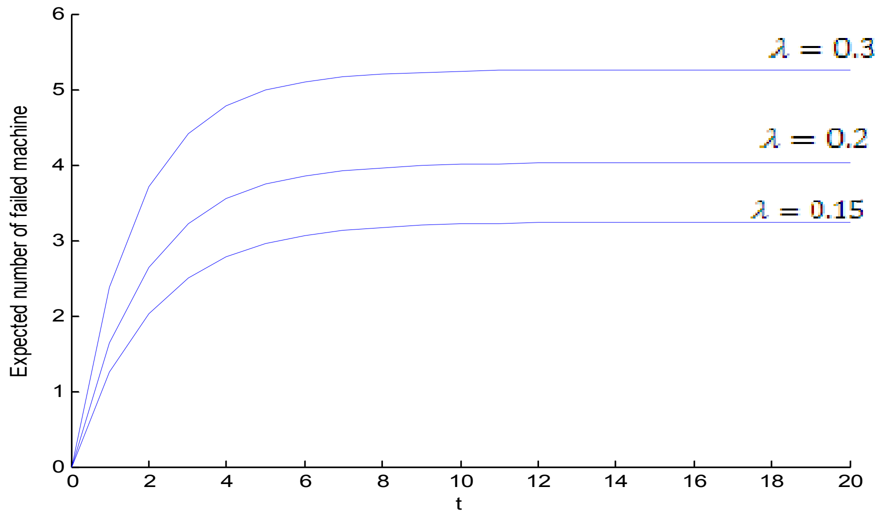

Fig. 3. The effect of failure rate of machines on the expected number of failed machine in the system at time t when μ=0.6, θ=1, α=0.05, β=10, M=10

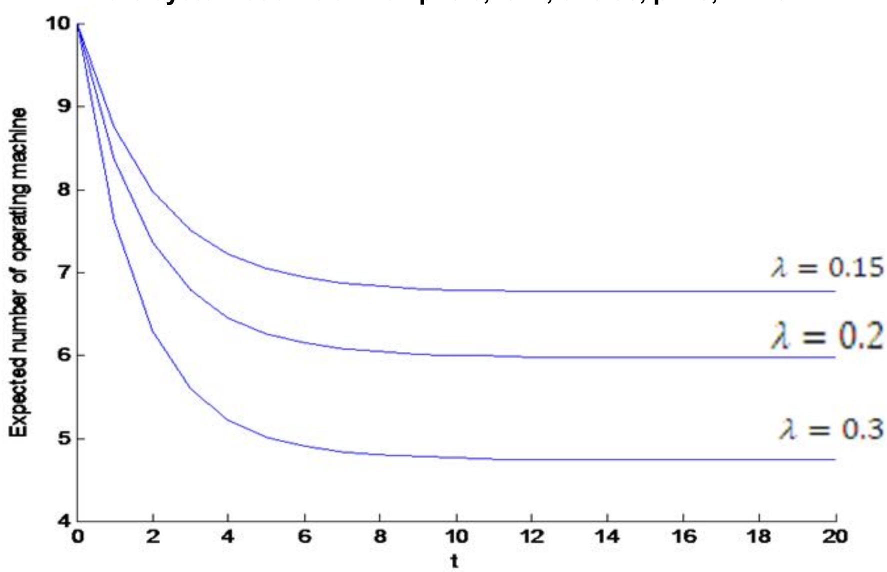

Fig. 4. The effect of failure rate of machines on the expected number of operating machine in the system at time t when $\lambda = 0.15, \theta = 3, \alpha = 0.05, \beta = 10, M = 10$

Fig. 5 shows the effect of breakdown and repair rate of server on the expected number of failed machine. We found that with the same breakdown rate and increased repair rate of server the expected numbers of failed machine decreases. And with the same repair rate β of broken down server α with increase breakdown rate of server the expected number of failed machine increases E(F). Examples of these results are shown in Table A.

Figs. 5 and 6 correspond to the result in Tables 8 and 9 above.

Table A. Expected number of failed machines for the different values of broken down server rate α and repair rate of server β

α	β	E(F)
$\alpha = 0.3$	$\beta = 5$	3.3802
$\alpha = 0.3$	$\beta = 10$	3.2617
$\alpha = 0.05$	$\beta = 5$	2.7176
$\alpha = 0.05$	$\beta = 10$	2.6952
$\alpha = 0.01$	$\beta = 5$	2.6095

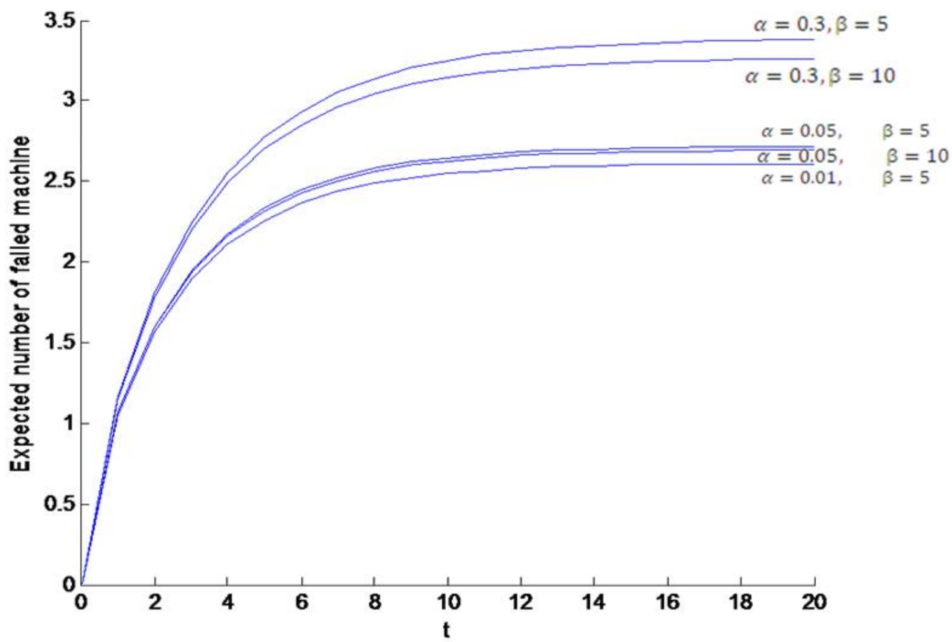

Fig. 5. The effect of breakdown and repair rates of server on the expected number of failed machine in the system at time t when $\lambda = 0.15, \theta = 3, M = 10$ $\mu = 1.1$

Fig. 6 shows the effect of breakdown and repair rate of server on the expected number of operating machine. We found that with the same breakdown rate and increased repair rate of server the expected numbers of operating machine increases. And with the same repair rate of broken down server with increase breakdown rate of server the expected number of operating machine decreases. Examples of these results are shown in Table B.

Table B. Expected number of operating machines for the different values of broken down server rate α and repair rate of server β

α	β	E(O)
$\alpha = 0.05$	$\beta = 5$	7.2824
$\alpha = 0.05$	$\beta = 10$	7.3048
$\alpha = 0.01$	$\beta = 5$	7.3905
$\alpha = 0.3$	$\beta = 10$	6.7383
$\alpha = 0.3$	$\beta = 5$	6.6198

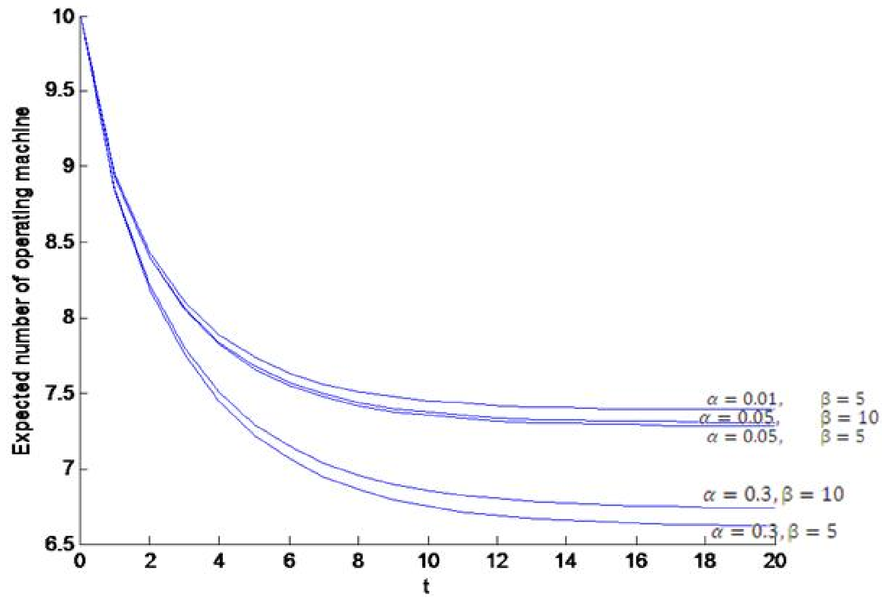

Fig. 6. The effect of breakdown and repair rates of server on the expected number of operating machine in the system at time t when λ=0.15, $\theta = 3$, M=10 $\mu = 1.1$.

Figs. 7 and 8 correspond to the result in Table 10. Fig. 7 shows the effect of vacation length on the expected number of failed machine in the system. We found out that as vacation length θ of server decreases the expected number of failed machine increases.

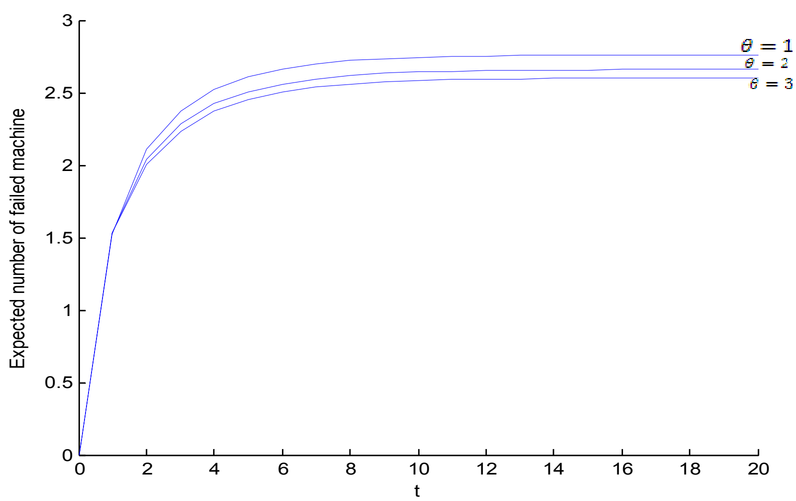

Fig. 7. The effect of vacation length on expected number of failed machine in the system at time t. When $\lambda = 0.15, \theta = 3, M = 10$ $\mu = 1.1$

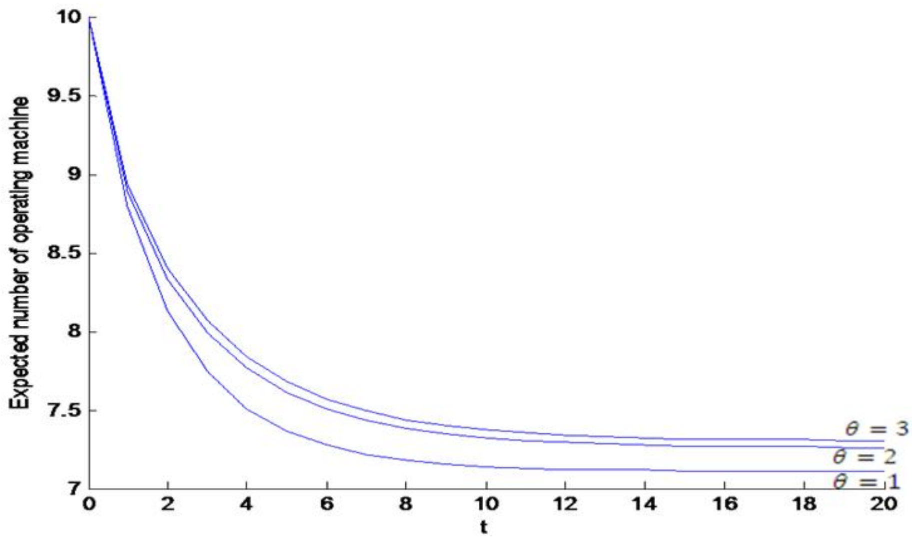

Fig. 8. The effect of vacation length on expected number of operating machine in the system at time t. when λ=0.15, $\theta = 3$, M=10 $\mu = 1.1$

Fig. 8 shows the effect of vacation length on the expected number of operating machine in the system. We found out that as vacation length increase the expected number of operating machine increases. This show that as the expected number of operating machine increases in the system the vacation length of the server also increases.

Table 11. Comparing the multiple vacations policy with the single vacation policy of Ojobor and Omosigho (2014) using the parameters $\lambda = 0.15, \mu = 1.1, \alpha = 0.05, \beta = 10, \theta = 1.0$ and $M = 10$

	Multiple vacation	Single vacation
E(F)	2.9879	2.8195
E(o)	7.0121	7.1805
E(v)	0.1752	0.1467
E(I)	0.0000	0.0391
E(D)	0.0046	0.0047
M.A.	0.7012	0.7181
O.U.	0.8202	0.8096
Var	0.5096	0.4874
STD	0.7139	0.6982

For the multiple vacations policy there are1+3M equations in the system, we observe that for small M says $M = 100$, the CPU time is also less than 2 seconds Table 12. We also observe that there is a relationship between the numbers of machine in the system and the CPU time. We use linear regression in EXCEL package to compute the predicted CPU time for the system. We found that the predicted $CPU\ time\ = a + bM$ where a and b are constants and M is the number of machines. We observe that the predicted CPU time is an indication of the actual CPU time.

Table 12. Effect of M on the machine availability and CPU Time for sufficient value of t for the multiple vacations. $M = 10, \lambda = 0.15, \mu = 1.1, \theta = 1, \alpha = 0.05, \beta = 10$

M	10	20	30	40	50	100
$E(F)$	2.9879	8.2134	14.1752	20.1472	26.1197	55.9830
$E(O)$	7.0121	11.7866	15.8248	19.8528	23.8803	44.0170
$E(v)$	0.1752	0.0051	0.0000	0.0000	0.0000	0.0000
$E(I)$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$E(D)$	0.0046	0.0050	0.0050	0.0050	0.0050	0.0050
$M.A.$	0.7012	0.5893	0.5275	0.4963	0.4776	0.4402
$O.U.$	0.8202	0.9900	0.9950	0.9950	0.9950	0.9950
Var of E(O)	0.5096	4.4781	14.2787	29.7167	50.7724	240.4693
ACTUAL CPU TIME(secs)	1.3427	1.3652	1.4268	1.4754	1.5343	1.6305
PREDICTED CPU TIME(secs)	1.3584	1.3913	1.4241	1.4569	1.4898	1.6540

We also found that as the number of operating machines increases the machine availability decreases while the operative utilization increases and become stable from $M = 30$. We also found that as the number of operating machines increases the CPU time to run the algorithm is higher in the multiple vacations policy than that of the single vacation policy. We also notice that the CPU time to run the algorithm is slightly high in the multiple vacations policy than that of the single vacation policy this is because the server has to undergo several vacations in the multiple vacations policy.

4 Conclusion

In conclusion, we considered transient state of the machine interference problem with an unreliable server under multiple vacations.

Using ODE45 (Runge-Kutta algorithm of 4 and 5 order for solving ordinary differential equations) in MATLAB package to solve the Chapman-Kolmogorov differential equations derive for the system, we obtain the transient probabilities for the system. From the transient probabilities obtained for each of the system we compute the expected number of failed machines, the expected number of operating machines and the machine availability with respect to time t. Apart from finding the transient result for the expected number of failed and operating machine given by Ke (2006) steady state result, we also obtain the variance of the expected number of failed and operating machines in the system for the multiple vacation which previous authors did not obtain in literature. We showed numerical results for the effect of the different parameters on the availability of the machine in the system. We also compared our results with existing ones. We also compared the operational measures of performance for the single and multiple vacations for the same parameters. We observed there is a slight different in the operational measures of performance for the multiple vacations and that of the single vacation with time t. This is because the servers have to undergo several vacations in the multiple vacations. The expected idle period in multiple vacations is zero. This is so because the server is not idle in multiple vacations. The server will continue to take vacations until there is one failed machine in the system. We also report on the CPU time for obtaining the transient results for each of the systems and the variance of the systems.

Competing Interests

Author has declared that no competing interests exist.

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APPENDIX

MATLAB code-solver

```
tic
clear all
global M
M = 10;
y0 = zeros(3*M+1,1);
y0(1)=1;
T1=[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20];
T=length(T1);
[t, y]=ode45('multiplevacation',T1, y0);
for i=1:T;
  y(i, :);
end
for i=1:T;
  sum(y(i, :));
end
% Normalization
for i=1:T;
 p(i, :)=y(i, :)/sum(y(i, :));
end
 for i=1:T;
     sum(p(i, :));
 end
 Expectednooffailedmachine(1)=0
 Expectednoofoperatingmachine(1)=M-Expectednooffailedmachine(1)


sum1=0;
sum2=0;
sum3=0;
for i=1:T
for j=1:M+1;
    sum1=sum1+p(i, j)*(j-1);
end
for j=M+2:2*M+1;
    sum2=sum2+p(i, j)*(j-(M+1));
end
for j=2*M+2:3*M+1;
    sum3=sum3+p(i, j)*(j-(2*M+1));
end
Expectednooffailedmachine(i)=sum1+sum2+sum3
Expectednoofoperatingmachine(i)=M-Expectednooffailedmachine(i)
Machineavailability(i)=1-(Expectednooffailedmachine(i)/M)
sum1=0;
sum2=0;
sum3=0;
end

 sum4=0;
 sum5=0;
```

```
for i=1:T;
    for j=1:M+1;
    sum4=sum4+p(i, j);
end
     for j=2*M+2:3*M+1;
    sum5=sum5+p(i, j);
      end
   Expectednoofvacation(i)=sum4
   Expectedbrokenserver(i)=sum5
   Expectedidleperiod(i)=0
   Expectedbusyperiod(i)=1-Expectedbrokenserver(i)-Expectedidleperiod(i)-
Expectednoofvacation(i)
   sum4=0;
   sum5=0;
end
    v=var(Expectednooffailedmachine)
    u=var(Expectednoofoperatingmachine)
    std=sqrt(v)
    toc
```


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