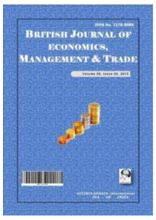

British Journal of Economics, Management & Trade
16(3): 1-17, 2017, Article no.BJEMT.31353
ISSN: 2278-098X

SCIENCEDOMAIN *international*
www.sciencedomain.org

SCIENCEDOMAIN

Measuring Real Equilibrium Interest Rate: From the Linear Equation of the Taylor Rule to Agent - Based Model

Alexander Dedishchev[1,2*]

[1] *Department of Economics, University of Vienna, Vienna, Austria.*
[2] *Department of Finance and Banking, Russian Presidential Academy of National Economy and Public Administration, Higher Educational Institution in Moscow, Russia.*

Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/BJEMT/2017/31353
Editor(s):
(1) Chiang-Ming Chen, Department of Economics, National Chi Nan University, Taiwan.
Reviewers:
(1) Victor Gumbo, University of Botswana, Gaborone, Botswana.
(2) Achille Dargaud Fofack, Cyprus International University, Turkey.
(3) Imre Ersoy, Marmara University European Union Institute, Turkey.
(4) Naoyuki Ishimura, Chuo University, Japan.
Complete Peer review History: http://www.sciencedomain.org/review-history/18112

Received: 1st January 2017
Accepted: 22nd February 2017
Published: 9th March 2017

Original Research Article

ABSTRACT

The aim of this paper is to transform standard Taylor rule linear equation model into agent-based computational (ABC) model. ABC model addresses monetary macroeconomics as a complex evolving system of heterogenous agents whose interactions continuously change the structure of the system.
Our ABC methodology extend the Taylor rule-based model with macroeconomic and global factors that account for the variability of the equilibrium interest rate and for the deviations of the actual federal funds rate from the interest rates implied by the Taylor rule.
As a result we compared two models and identified that ABC model allows to better understand forward looking equilibrium interest rate dynamics comparatively to Taylor's mathematically determined 2% steady state equilibrium interest rate.

Keywords: Agent based model; monetary economics; Taylor rule; equilibrium interest rate; global demand for reserve currency; financial imbalances; wealth velocity of money.

**Corresponding author: E-mail: adedic@yandex.ru;*

1 INTRODUCTION

The global economy is a complex and dynamic system. The macroeconomic equilibrium of money supply and money demand desired by monetary authorities changes over time. These changes result from the frequent shocks that affect the equilibrium of the economy and the corresponding equilibrium interest rate.

The Global Financial Crisis and the Great Recession created new objectives for monetary authorities around the world. A new economic reality requires a wider range of agents to be integrated into existing models. Blanchard (2014) [1] claimed that traditional models are useless when the conditions of the Great Moderation do not hold.Every Central Bank can stop inflation, but it is not possible to halt a deflationary process with old methods of monetary manipulation. Growing global financial imbalances bring new challenges to the central banks.

In order to better understand interdependencies we have developed a priori set of multiple regression models based on the existing theories and on the empirical research. The goal of this working paper is to enhance our understanding of the real equilibrium interest rate behavior based on agent-based models.

The new economic reality requires new approach to manage inflation, growth and stability. The aim of this research is to study the interaction between federal funds rate, equilibrium interest rate, macroeconomic variables and global factors in an agent-based modeling framework.

This working paper proposes a paradigm that can escape strong theoretical requirements of neoclassical, New-Keynesian, ABC and other theoretical models and that also avoids the constrains of rational expectations. It embraces an evolutionary perspective, according to the definition of G. Fagiolo and A. Roventini (2016) [2]. They claim that existing DSGE, VAR, SVAR and GVAR models are hard to solve within neoclassical and New Keynesian theories.

John Taylor described preferred policy rules that have emerged from his research: "The rules are responsive, calling for changes in the money supply or short term interest rate in response to changes of the price level or income" [3]. This paper further develops the idea that the supply of U.S. dollars balances a global demand for reserve currency and that understanding of the real equilibrium interest rate is an important tool to balance out supply and demand. We examine the role of critical factors that shifted the demand for reserve currency. We estimate the influence of these shocks by multivariate time series models, which include exogenous and endogenous factors to capture these influences.

This paper further develops the agent-based validation method developed by M. Guerini and A. Moneta (2016) [4], which validates simulations models that generate artificial time series comparable with real-world data. We base our empirical part of the research on Structural Vector Autoregressive (SVAR) model.

Given the complexity of the proposed method we start with the empirical identification of the relationship between variables and dimensions of macroeconomic activity, in the first place: prices, inflation, employment and monetary aggregates. Thereby we replicate statistical properties of the variables. Preliminary analysis showed that there is no direct effect of monetary policy on inflation and output.

This paper, by identifying and comparing causal structures, proposes a method that improves the empirical reliability of the policy-oriented simulation models.Based on empirical results we argue that relying on a relatively small range of variables does not permit the identification of the complete dynamics of the influencing factors. These factors drive fluctuations of the equilibrium interest rate and as a result influence the federal funds rate formation, as the federal funds rate directly depends on the real equilibrium interest rate.

Linking Structural Vector Autoregression capabilities to agent-based models allows to reduce complexity and improve efficiency of the agent-based model (ABM).

2 DATA AND METHODOLOGY

We use quarterly U.S. time series data over the period 1983:Q1–2015:Q1.12. In order to induce linearity we applied the logarithm function to the Federal Debt Hold by Foreign Investors (log_{FDFI}) and to the Wealth Velocity of Money (log_{WVM}). Transforming other variables did not appear beneficial. The quarterly data of the output gap is based on real GDP and potential GDP, inflation gap on real inflation and desired long–run inflation rate goal. The policy rate r is measured by the federal funds rate. The source of the data: Bureau of Labor Statistics http://www.bls.gov/, Bureau of Economic Analysis of U.S. department of Commerce https://www.bea.gov/, Federal Bank of St. Luis https: //research.stluisfed.org/ and Institute of International Finance IIF https://www.iif.com

In the first step we break the model into static and dynamic tiers. Then we test the model for serial correlation using Ljung – Box Q statistic [5]. We use augmented Dickey Fuller Unit Root method to test [6] the time series for a unit root against a trend-stationary alternative augmented with lagged difference terms.

We use algorithm to estimate the reduced form Vector Autoregression (VAR) of equation, then we identify SVAR (Structural Vector Autoregression) using computationally efficient algorithm designed for restrictions imposed on impulse responses developed by J. Rubio Ramirez et al. [7]. After identification stage we apply a search algorithm to the estimated residuals and then empirically validate the model by showing that the statistical properties of simulated macroeconomic data are in line with the empirically observed ones. This process allows us to provide evidence that agent-based model is an adequate representation of the real-data. The final step is results transformation of the preparatory work into the simulation framework. For the simulation will use AnyLogic System dynamics modeling method that enables us to build computer simulations of complex systems and use them to design effective monetary policy. As a conclusion we compare results of the simulation based on the mathematical approach and on the causal search algorithms. For this purpose we use powerful Akaike Information Criteria (AIC). AIC was developed by Akaike (1973)[Akaike1973] to compare different models influence on a given outcome.

3 SETTING THE SCENE: WHAT IS THE EQUILIBRIUM INTEREST RATE PHENOMENON?

The Taylor rule revolutionized the monetary policy of central banks. The rule has become an important instrument of response to any deviations from potential GDP, desired inflation and "natural unemployment". The rule, which is based on US experience and statistics, prescribes the following formula:

$$r \ = r^{*} + \pi_t + \beta(\pi_t - \pi t^{*}) + \beta 1(yt - yt^{*}),$$

where r represents the recommended federal funds rate, r^* represents the real equilibrium real interest rate, $\pi_t - \pi_t^*$ is the deviation of inflation from desired long–run inflation rate goal and $y_t - y_t^*$ represents the difference between actual real output and potential output.

Since the beginning of the financial crises, there has been a great deal of research and debate on estimating the real equilibrium interest rate that is consistent with full employment and stable inflation. The real equilibrium interest rate continues to be in the center of discussions about economic policy in the whole world. According to the Financial Times Ben Bernanke written that: "the real equilibrium interest rate, and not the FOMC, is the ultimate determinant of interest rates of the economy". He claims that the real equilibrium interest rate is discussed at every Fed morning[8].

The existing literature explains the reduction of the real equilibrium interest rate by the correlation between trend growth rate and the real equilibrium interest rate. This topic has been thoroughly studied by Williams (2015) [9] and by Hamilton et al. (2015) [10]. Laubach and Williams (2003, LW) based their estimation of the real equilibrium interest rate on Taylor's assumption that the equilibrium interest rate depends on the trend growth rate of potential output. This trend dependence, however, ignores the evidence of the fundamental exogenous factors. Moreover, with the trend growth rate as a regressor variable, the original substantial effects of other variables disappear. In the original essay on the Taylor rule (Taylor, 1993) [3], John Taylor opined that the equilibrium interest rate of 2 % was close to the steady-state growth of 2.2%, whereas more recent evidence indicates a substantially lower equilibrium rate. The Taylor rule does not capture the global influence on the equilibrium interest rate. The trend growth rate proxies factors that should be revealed in order to understand the causes of the reduction in the real equilibrium interest rate.

4 EMPIRICAL ESTIMATION OF THE RELATIONSHIPS BETWEEN TAYLOR RULE VARIABLES

The aim of the first step is to reveal factors, which from the end of the last century have accounted for systematic deviations from the Taylor rule relationship between real equilibrium interest rate, federal funds rate, inflation and output. Using the models tier structure proposed by C. Sims in his working paper "The role of models and probabilities in the monetary policy process" [11] we break the model into static and dynamic tiers. The static regression specification does not admit any clear causal interpretation. We start our analysis by including federal funds rate as predictor at times t-1 and t-2. The results of the regression are summarized in Table 1.

The regression analysis does not show much evidence for the success of U.S. monetary policy. The characteristics of the statistical model in Table 1 support the hypothesis that changes in the predictor are associated with changes in the response, which indicates that the entire model is statistically significant. According to the characteristics of the model, the one-year lagged federal funds rate variable has a genuine effect on unemployment. We have little confidence in the results, however, as the one quater lagged federal funds rate regression coefficient is negative, which appears counterintuitive. Even with statistical significance of the lagged variable it is difficult to explain the negative coefficient, as the Federal Reserve believes that lower federal funds rate stimulates employment. An explanation may be that the causal direction runs from the dependent variable toward the regressor. The federal funds rate may have been set at a lower value if unemployment is on the rise, to fight recessionary tendencies by a monetary expansion.

The one-year lagged variable has power of prediction in spite of having no sensible effect on unemployment. The second lag is statistically insignificant. Therefore we should reduce the number of lagged predictor variables in order to build a separate predictor model. Regression statistics for the period from 1993 to 2015.

Table 1. Regression of unemployment on the FFR with two-quarter lags

Dependent variable: Unemployment Rate				
Number of cases: 80				
$R2$: 0.597				
Adjusted R2: 0.586				
Explanatory Variable	coefficient	std. err.	t–stat	p-value
Intercept	7.465	0.190	39.347	0.000
Federal funds rate (t–1)	-0.785	0.299	-2.620	0.011
Federal funds rate (t–2)	0.220	0.298	0.737	0.464

In Table 2 we consider the FFR lagged by one quarter. The federal funds rate influences the output growth rate with a regression coefficient of 0.058, which is a low impact on the growth of the economy output. The small corresponding p–value of 0.071, however, clearly supports the significance of this relationship. But p–value indicates that the estimated model is significant at the generous 10 % level only. The small R of 0.043 again demonstrates that only an insubstantial portion of the variation in output can be explained by monetary policy. We have analyzed short-term influence of the monetary policy on the output. The statistical model with the one-quarter lagged Federal Funds Rate variable also reveals that the increase or decrease of the Federal Funds Rate does not stabilize the economy in a short nor in a longer term. Models with two quarters and three quarters lagged Federal Funds Rate variables are even less meaningful. The Taylor rule, however, can be a robust mechanism in a closed economy.

The statistical analysis in Tables 1 and 2 reveals that, during the last 20 years, monetary policy has had little direct effect on GDP. The low R^2 in both specifications indicates that the monetary variable has had no tangible impact, neither on inflation nor on output.

Regression statistics for the period from 1994 to 2016.

Controlling inflation is the most important responsibility of Federal Reserve. Therefore it needs to know in advance when inflation is most likely to increase or to decrease. As a next step we will regress inflation on its own past values. This simple auto regression captures tendencies correctly: within analyzed period of 236 quarters inflation gradually returns to its initial level before external shocks. We expect past inflation having positive effect on future inflation. Both coefficients p_{t-1} and p_{t-2} have positive signs. The inclusion of autoregressive lags considerably improves statistical significance of the model. The data become more informative about interest rate persistence. Regression statistics for the period from 1996 to 2016.

Figure 1 shows that in this period inflation ran below or higher than predicted by Taylor rule and targeted by the Federal Reserve, especially after the financial crises in 2008 when rates were sluggishly low. Cook and William (2015) [12] conducted empirical research and justified that zero interest rate policy should be treated “as a separate regime with different statistical processes than observed in the United States during the Great Moderation”. Globalization, zero interest rate policies and disbalances of the world economy altered variables responsible for stable growth and employment rate.

Table 4 shows the outcome from a model in which we reduce the number of lags to one. Here only one value seems to matter – the rate of inflation lagged one year. Having clearly identifiable effect we can assume what form of the model is most appropriate for inflation. However, in light that quarterly autoregressive coefficient is close to 1, we need to check stationarity of the model and it would be of interest to find exogenous factors influencing inflation.

Table 2. Regression of GDP growth on the FFR with a one-quarter lag

Dependent variable: quarterly real GDP growth

Number of cases: 77
R^2: 0.043
Adjusted R^2: 0.030

Explanatory Variable	coefficient	std. err.	t–stat	p-value
Intercept	0.433	0.114	3.795	0.000
Federal Funds Rate$_{(t-1)}$	0.058	0.032	1.834	0.071

Table 3. Regression of inflation on its own two quarterly lags

Dependent variable: Inflation Rate				
Number of cases: 236				
R^2: 0.784				
1-1 Adjusted R^2: 0.782				
Explanatory Variable	coefficient	std. err.	t–stat	p-value
Intercept	0.060	0.032	1.236	0.234
$p_{(t-1)}$	0.557	0.061	13.123	0.000
$p_{(t-2)}$	0.364	0.061	0.060	0.953

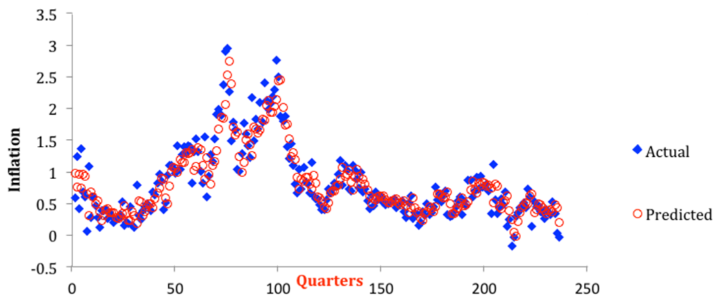

Fig. 1. Prediction of Inflation with autoregressive lags

Table 4. Regression of inflation on its own one-quarter lag (1996 - 2016)

Dependent variable: Inflation Rate				
Number of cases: 236				
R2: 0.752				
Adjusted R2: 0.751				
Explanatory Variable	coefficient	std. err.	t–stat	p-value
Intercept	0.102	0.033	3.096	0.002
Federal Funds Rate (t-1)	0.871	0.033	26.606	0.000

5 INFLUENCE OF THE EMERGING INDEPENDENT VARIABLES ON THE EQUILIBRIUM INTEREST RATE

The statistical analysis of this paper reveals factors that influence the equilibrium interest rate. These factors are: federal debt held by foreign investors, the current account balance, and the growth of wealth. The equilibrium interest rate accommodates to the changing liquidity demand in the global environment and to the policies of central banks.

We start our analysis with the determination of a relationship between the real equilibrium interest rate as dependent variable and the global demand for reserve currency, financial imbalances, and growth of wealth as explanatory variables. The regression model reported in Table 5 reveals the influence of several variables on the equilibrium interest rate: Federal Debt Held by Foreign Investors, inflation gap, the Current Account balance as per cent of GDP, and real equilibrium federal funds rate lagged one year.

Table. 5. Regression of the equilibrium real interest rate on influencing factors for the period from 1985 to 2016

Dependent variable: Real Equilibrium Federal Funds Rate.				
Number of cases: 31				
R^2: 0.884				
Adjusted R^2: 0.866				
Explanatory Variable	coefficient	Std. Err.	t-stat	p-value
Intercept	2.755	0.999	2.758	0.010
log FDFI[1]	-0.853	0.302	-2.83	0.009
Inflation Gap	0.156	0.101	1.547	0.134
Ratio of Current Account to GDP	-0.22	0.059	-1.592	0.123
$r^*_{(t-1)}$	0.521	0.146	3.528	0.001

[4]FDFI - Federal Debt held by Foreign Investors

The model has a strong predictive power as indicated by R^2 of 0.884. This indicates that almost 90% of the total variance is explained by the model. These revealed variables predict a low real equilibrium interest rate. In order to induce linearity we applied the logarithm function to the Federal Debt held by Foreign Investors . Transforming other variables did not appear beneficial. The regression model has a coefficient estimate of –0.853 for the logarithm of the Federal Debt held by Foreign Investors. This variable is significant for all specifications. By contrast, the inflation gap has no statistically significant impact on the real equilibrium interest rate. The ratio of the Current Account balance to GDP, however, shows a strong influence on the low real equilibrium interest rate.

The regression coefficients on the log-FDFI and on the ratio of the Current Account to GDP show negative signs. The sizable negative correlation between global misbalances and the real equilibrium interest rate supports the argument that the United States cannot simultaneously stabilize wage inflation, reduce their current account deficit, and satisfy worldwide demand for reserve assets.

The global economy and the growth of wealth require expanding the supply of currency abroad by running current account deficits. Therefore, our statistical analysis confirms that globalization has reduced the sensitivity of domestic inflation and domestic output to domestic monetary policy. Such negative signs are interpreted as disrupting factors for interest rate stability. These facts complicate the task of monetary policy. The analyses suggest that the Taylor rule is not appropriate for conducting monetary policy in an environment of global financial misbalances and “excess elasticity”. Borio and Disyatat (2011) introduced the related concept of “excess financial elasticity ”[13]. Chung K et al. (2014) in their IMF working paper furthermore researched this subject and highlighted the importance of trade balances in measuring money aggregates. “Our observations on the cross-border activity of firms are relevant for the discussion of measuring liquidity aggregates”[14].

Aggressive monetary policy with excess financial elasticity contributes to the build-up of financial misbalances in the world economy, and an excess demand for safe assets exacerbates this process. In 2005 Ben Bernanke and K. Kuttner [15] claimed that: “The most direct and immediate effects of monetary policy actions, such as changes in the federal funds rate, are on the financial markets by affecting asset prices and returns. Policymakers try to modify economic behavior in ways that will help to achieve their ultimate objectives. Understanding the links between monetary policy and asset prices is thus crucially important for understanding the policy transmission mechanism.” In 2004 Bjrnland and Leitemo come the conclusion that stock prices may influence consumption through wealth effects and influence investments through the Tobin Q effect [16].

As the next step of this research I address gaps in existing empirical studies to explain the nature of current account balances and lack of desired inflation after many years of aggressive monetary policy.

The growing amount of currency held by public and corporations plus growing banking reserves complicate this tendency. Rogoff (2003) [17] argued that globalization played an important role for disinflation. The U.S. liquid financial market and the growing public debt allow the U.S. government to be a provider of safe assets.

This paper identifies a link between monetary policy, the inflation of financial assets and the growth of wealth. The correlation between the money stock and the growth of wealth is tentatively causally interpreted as describing the reaction of the growth of wealth to money supply. This research further develops Milton Friedman's restated (refined) Modern Theory of Money. According to this theory, money demand (and consequently velocity of money) depends on income, interest rate and total wealth. This means that an increase in total wealth results in an increase in the total demand for money and as a result in the wealth velocity of money.

Santoni (1987) has introduced the concept of wealth velocity of money, motivating that the quantity of money is related more to the households' wealth than to their income, and that money is simply one of the many assets in which the wealth may be held. This theory suggests that the demand for money is a constant proportion of wealth. The ratio of wealth (W) to money (M3) is called the Wealth Velocity of Money. The theory suggests that wealth consists of five elements: money, bonds, shares, material goods and human capital.

In the words of Milton Friedman (1969) [18]: "The demand for money (or any other particular asset) depends on three major sets of factors: (a) the total wealth to be held in various forms; (b) the price of and return on this form of wealth; and (c) the tastes and preferences of the wealth-owning units and risk profile". According to this theory, the increase in wealth is associated with an increase in the amount of money people want to hold. In order to satisfy this demand U.S. dollars are widely used in international transactions, in foreign exchange markets, and for the accumulation of wealth. Results of this statistical research substantiate the portfolio wealth theory that money is only a one way to store wealth.

According to Carpenter and Lange (2002) [19] demand for financial assets is sensitive to changes in the interest rate. This work will further empirically develop liquidity preference theory that speculative demand for money is highly elastic to low interest rate.

Belke and Polleit[20] in their book presented formula for wealth velocity of money, which takes into account personal wealth, such as bonds, equities, real estate and etc. They expressed the role of wealth for the velocity of money aggregate M3:

$$Stock\ of\ money = MV_{YW} = \left(Y^{\alpha} \text{x}\ S^{1-\alpha}\right) * \left(P_Y^{\beta} P_F^{1\beta}\right), \quad (5.1)$$

where:

$V - Income\ and\ Wealth\ Velocity\ of\ Money\ \ (all\ money\ ransactions)$

$Y - GDP$

$S\ - Personal\ Wealth$

$P_L - Price\ level\ of\ GDP$

$P_F - Price\ level\ of financial\ assets$

$\alpha\ - share\ of\ GDP\ transactions\ in\ total\ transaction\ volume$

$(1-\alpha\ ) - share\ of\ financial\ transactions\ in\ total\ transaction\ volume.$

We transform variables into logarithms and regroup the formula 1:

$\nu_{YW} = \alpha y + (1\alpha)s + \beta p_Y + (1\beta)p_F m.$

Ratio $\frac{Y}{W}$ reflects change of GDP relative to Wealth and therefore wealth velocity of money to income velocity of money. Further we use this ratio to calculate relationship between wealth velocity of money relative to income velocity of money.

6 TESTING FOR SERIAL CORRELATION AND STATIONARITY

As a next step we follow the traditional approach of selecting the necessary amount of lags and testing for serial correlation in order to exclude the possibility of dynamic misspecification. We add lagged versions of our dependent variable as regressors in order to analyze whether autocorrelations of a time series of equilibrium interest rate differ from 0.

New Keynesian theory assumes that monetary authorities should adjust policy rate one-for-one with the real equilibrium interest rate, thus assuming observability of the latter variable. But determination of the real equilibrium interest rate and its movements requires an exact knowledge of the economy's true model. In further analysis we use dynamic model for the estimation of the equilibrium interest rate linking it to the lagged equilibrium interest rate, expected growth rate and to the federal debt held by foreign investors, the variable, which is the proxy of globalization factors.

We decided to add a lagged version of our dependent variable as an independent variable in our model, because we have concerns that our model would be biased otherwise. We add lagged versions of our dependent variable as an independent variables in order to analyze weather auto correlations of a time series of equilibrium interest rate are different from 0. We compute p-value of the statistical portmanteau Ljung-Box test[5].

Portmanteau test statistic was presented by Box and Piers:

$$Q_{BP} = n\sum_{k=1}^{m} r_k^2, \; where$$

r_k is the sample autocorrelation of number k of the residual, m is the number of autocorrelations calculated (equal to the number of lags specified). In order to address finite sample distribution of the test statistic proposed and its conservative behavior Ljung and Box in 1978 introduced a portmanteau test on the basis of r_k. Ljung and Box formula tests randomness against a serially correlated series of several lags:

$$Q_{LB} = n(n+2)\sum_{k=1}^{m}(n-k)^{-1} r_k^2, \; where$$

n is a length of time series.

Thus we will examine if the predicted model is suffering from serial correlation and heteroskedasticity. We can see from descriptive statistics that residuals are auto correlated.

Table 6. Descriptive statistics of Ljung-Box white-noise test (WNTest) for equilibrium interest rate

Descriptive Statistics				
Average: 2.44916				
STD DEV: 1.				
Skew: -0.57				
Significance Test	Target	p-value	SIG	
	0.000	0.00%	True	
	0.000	0.04%	True	
White-noise Test	Score	C.V.	p-value	Pass?
Lag 1	213.07	3.84	0.0%	False
Lag 2	420.00	5.99	0.0%	False
Lag 3	620.63	7.81	0.0%	False
Lag 4	814.84	9.49	0.0%	False
Lag 5	1002.49	11.01	0.0%	False
Lag 6	1183.48	12.59	0.0%	False
Test	p-value	SIG		
White noise	0.00%	False		
Normal distributed?	0.22%	False		
Arch effect?	0.00%	True		

In a statistical sense equilibrium federal funds rate time series is characterized by no white noise, which implies serial autocorrelation. Equilibrium federal funds rate may be serially correlated as errors capture the influence of globalization shocks beyond output gap and inflation gap. Portmanteau test above shows that starting from 2001 until 2015 error term and independent lagged variables show correlation, which means that there might be omitted variable bias.

Results of Augmented Dickey Fuller Unit Root Test shows that our model of equilibrium interest rate is non-stationary. We achieved stationary through detrending and differencing. Results in Table 7 show that statistic value is lower than critical values at 1%, 5%, and 10% levels. As a result we reject the null at conventional levels. The first difference of the equilibrium interest rate time series is a stationary process.

Table 7. Equilibrium interest rate unit root test

		t - Statistic	Prob*
Augmented Dickey Fuller Unit Root Test	1% level	-13.480425	0.000
	5% level	-2.883408	
	10% level	-2.578510	

*MacKinnon (1996) one sided p-values

We assume that long-run trend and short-run cycles are built upon the assumption that the trend component is a deterministic function of time, and the cyclical component represents a stationary movement around this trend. This assumption does not hold if the trend component is non-stationary.

7 FROM MULTIPLE REGRESSIONS TO AGENT-BASED MODELS

7.1 SVAR identification

Given the stochastic nature of the most agent-based models, we draw outcomes and descriptive statistics from numerous runs of multiple regressions. We realized sufficient number of regression runs in order link multiple regression capabilities with the learning techniques of agent-based model. We start designing agent-based model with SVAR identification procedure. Then we incorporate casual structure in the artificially generated model and in the model with the real data. Next we estimate and compare the model derived by the mathematical approach and the model with the incorporated causal structure.

For the simulation of shocks we use the Cholesky decomposition method. This method requires an ordering of the variables, so there will be different initial impulse responses depending on which ordering one uses. The ordering implies structural restrictions (assumptions) regarding contemporaneous effects of the variables. We use the Cholesky decomposition where covariance matrix Ω_R = AA and A is a triangular matrix. As we would like that the variances of shocks are not unity, we would allow of A to capture the standard deviations. In this case according to S. Ouliaris et al [21] the contemporaneous impulse responses to unit shocks will be given by A.

The correlations between shocks in our model take place due to contemporaneous correlations between variables and not depend on lagged values. When contemporaneous effects are captured the errors in structural equations can be considered as uncorrelated. In this case the system consists of structural equations.

The structural VAR system with p components can be presented as follows:

$$A_0zt = A_1z_{t-1} + \ldots + A_pz_{t-p} + e_t,$$

where e_t is a structural error and A is a coefficient matrix.

The shocks in this case are uncorrelated. It means that $E(e_t) = 0$; $cov(e_t) = \Omega_S$ and Ω_S is a diagonal matrix. A_0 matrix can be presented as follows:

$$A_0 = \begin{bmatrix} a_{11}^0 & a_{12}^0 & . & . \\ a_{21}^0 & a_{22}^0 & a_{23}^0 & . \\ . & . & . & . \\ . & . & . & . \end{bmatrix}$$

When we run real data SVAR model with the real equilibrium interest rate we will have more than two variables as in the example below. Based on our preliminary analysis we present SVAR instrument principle where equilibrium interest rate has negative correlation with current account. In our preliminary analysis we proved statistically that this relationship holds due to the fact that the most of the demand for US dollar, as a reserve currency, is satisfied via negative current account. This relationship is similar to the formula of the supply and demand for money in terms of interest rates.

$$r_t^* + a_{12}^0 q_t = a_{11}^1 r_{t-1}^* - a_{12}^1 q_{t-1} + e_{1t}$$
$$q_t + a_{21}^0 r_t^* = -a_{21}^1 r_{t-1}^* + a_{22}^1 q_{t-1} + e_{2t},$$
$$var(\varepsilon_{1t}) = \sigma_1^2; var(\varepsilon_{2t}) = \sigma_2^2;\ cov(\varepsilon_{1t}\varepsilon_{2t}) = 0,$$

where r* is an equilibrium interest rate and q is a US current account.

Thus we can reduce autoregression to the equations:

$$r_t^* = a_{11}^1 r_{t-1}^* - a_{12}^1 q_{t-1} + e_{1t}$$

$$q_t = -a_{21}^1 r_{t-1}^* + a_{22}^1 q_{t-1} + e_{2t}$$

These equations suggest the form:

$$A_0 = \begin{bmatrix} 1 & -a_{12}^0 \\ -a_{21}^0 & 1 \end{bmatrix}$$

In our SVAR model we introduce dynamic restrictions in the regression models and damp empirically irrelevant exogenous variables. We model all variables jointly instead of modeling regression equations separately.

We start off with a set of 28 variables and at the end of the process we will have a smaller number of variables divided into exogenous and endogenous variables. We augment the system by restricting the number of variables and placing restrictions on the contemporaneous matrix of structural parameters. In our model the domestic block of variables is the following: money aggregate M3, Net Worth of Households (NW), inflation gap (π_{gap}), output gap (GDP_{gap}) and growth of Gross Domestic Product (GDP_{growth}), logarithm of Wealth Velocity of Money (log_{WVM}), trend GDP ($trend_{GDP}$), while exogenous variables are: Current Account (CA), Logarithm of Federal Debt held by Foreign Investors (log_{FDFI}) and Current Account relative to GDP (CA/GDP) and some others foreign block variables. We also divide the variables into a monetary policy block of variables and a non-policy block. Thereby we can exactly identify the SVAR model.

We define shocks by removing all those components from the structural model that are known at t - 1 . By focusing on the relation below given by formula (7.1) we only focus on unexpected change in variables. This represents considerable difference to the traditional dynamic models.

The relation between monetary policy shocks and variables is given by:

$$e_t = \bar{\text{A}}\text{h}_t, \tag{7.1}$$

where h_t - structural disturbances (shocks), Ā - is the contemporaneous coefficient matrix in a structural form and e_t is a structural error.

We identified the following relationships:

1. Interest rate is deeply impacted by the world of outside shocks.
2. It is assumed that the growing demand for reserve currency and global misbalances affect all these domestic variables contemporaneously.
3. The transmission of international shocks: CA/GDP and $\log_{FDFI}$ to the equilibrium interest rate can be very rapid. For example, the negative current account as ratio to GDP and the logarithm of Federal Debt Hold by Foreign Investors results

in an immediate decrease in the real equilibrium interest rate.
4. The real equilibrium interest rate reacts contemporaneously to the households' net worth.
5. The real equilibrium interest rate responds contemporaneously to the Wealth Velocity of Money.

We use identification restrictions to ensure that identified restrictions produce distinct impulse responses in the economic variables.

The short-run restrictions are:

1. Aggregate demand shocks have no contemporaneous effect on the equilibrium interest rate.
2. The quantity of money supply M3 does not have contemporaneous effect on the real equilibrium interest rate.
3. The quantity of money supply M3 does not have contemporaneous effect on the households' net worth.
4. Another exception excludes an immediate impact of US GDP gap on inflation.
5. The real equilibrium interest rate does not respond contemporaneously to the output and changes in the domestic monetary policy.

The domestic variables are deemed not to affect the international variables. Price puzzle is the phenomenon by which expansionary monetary policy leads to the lower price level.

The long run restrictions on impulse responses are:

1. Monetary policy shocks have no long-run effect on GDP growth.
2. Aggregate demand shock (GDP) has no long-run effect on the real equilibrium interest rate.
3. Monetary policy shocks have no long-run effect on the households' net worth.
4. Global demand for reserve currency is uncorrelated with money supply M3.

$$\begin{bmatrix} \eta_{GDPgap} \\ \eta_{\pi gap} \\ \eta_{log_{WVM}} \\ \eta_{log_{FDFI}} \\ \eta_{CA} \\ \eta_{CA/GDP} \\ \eta_{GDPgrowth} \\ \eta_{trend_{GDP}} \\ \eta_{M3} \\ \eta_{NW} \\ \eta_{r^*} \end{bmatrix} = \begin{bmatrix} 1 & a_{12} & a_{13} & ... & 0 & 0 & 0 \\ 0 & 1 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 1 & ... & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & ... & 0 & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 \\ a_{61} & a_{62} & a_{63} & ... & 0 & 0 & 0 \\ 0 & 0 & a_{73} & ... & 0 & 0 & 0 \\ a_{81} & a_{82} & a_{83} & ... & 0 & 0 & 0 \\ 0 & a_{92} & a_{93} & ... & 1 & 0 & 0 \\ 0 & a_{102} & a_{103} & ... & a_{109} & 1 & 0 \\ a_{111} & a_{112} & a_{113} & ... & a_{119} & a_{120} & 1 \end{bmatrix} \begin{bmatrix} e_{GDPgap} \\ e_{\pi gap} \\ e_{log_{WVM}} \\ e_{log_{FDFI}} \\ e_{CA} \\ e_{CA/GDP} \\ e_{GDPgrowth} \\ e_{trend_{GDP}} \\ e_{M3} \\ e_{NW} \\ e_{r^*} \end{bmatrix} \quad (7.2)$$

The impact of the monetary policy on the inflation is almost completely offset by the growing demand for the researve currency. Monetary policy does not completely counteract deflation process, which is USA below baseline for about 7 years after the crises of 2008. For the domestic variables in general, the exogenous global variables are responsible for a large proportion of forecast errors, especially for the long lags.

7.2 Improving Identification - Applying Casual Search Algorithm

The next approach to SVAR identification is to incorporate casual structure in the artificially generated model and in the model with the real data. Then we estimate and compare the models derived by mathematical approach and the model with incorporated casual structure.

Under the second approach the identification procedure is based on casual search algorithm which estimates residuals e_t for the definition of matrix A_0. The main diagonal of the matrix is 1. This methodology was developed by M. Guerini and A. Moneta (2016) [4] and requires the following steps:

1. Estimate the reduced form of VAR equation.
2. Apply a search algorithm to the estimated residuals.
3. Recover the other matrices of the SVAR model.

The proposed search algorithm was introduced by Spirtes et al. (2000) [22]. This algorithm does not allow causal loops and has the following stages:

1. Make all necessary connections - make sure that each vertex connected to any other vertex.
2. Cut some edges.
3. Build colliders.
4. Direct some other edges UNTIL no more edges can be oriented.

The proposed algorithm requires input of conditional independence relationships among the variables for each set of economic parameter values. As the final step we identify the structural form of the model by means of the selected causal search algorithm.

It is very difficult in the environment of the global economy imbalances to model the equilibrium interest rate of the main reserve currency. This means that all international agents need to coordinate on the equilibrium path to the real equilibrium interest rates. Existing models fail to account for the demand of safety assets denominated in U.S. dollars and growth of wealth in United States of America.

Analysis of this working paper show that Monte Carlo simulations with agent-based data that is based not only on the exogenous factors, but also on the global factors and wealth variables match dataset of the real world data much better. An evolutionary model of the real equilibrium interest rate develops in line with the globalization and the accumulation of wealth. Contemporaneous dynamics of the real equilibrium interest rate are defined by the impact of the global demand for reserve currency and impact of wealth.

7.3 Agent-Based Simulation

After selecting appropriate variables by mathematical approach and casual search algorithm we create a dataset of the real world data (RW) and as well data from Monte Carlo simulations or Agent-Based data (AB-data)[4]. Given the selected variables we assign agents to interact and update knowledge about the environment. This patterns will be used to reproduce empirical knowledge. Then we introduce a new set of variables into the model for the next iteration. Each iteration of agents creates a new environment and values for variables. The model runs repeatedly and each time generates distribution of results. The estimation procedures are tested by the means of Monte Carlo simulations with the same number of observations as in the real data. We estimate agent-based model by using a scoring algorithm.

The model will simulate the effects of highly adverse shocks under different assumptions for the response of monetary policy and estimate the current dynamics of the economy, including sensitivity of the inflation to output, interaction between spending and production trough changes in financial conditions, including movements in financial wealth, expectations of firms and households about the future developments of short and long term interest rates and growing demand for research currency.

Financial wealth, expectations to interest rate developments (yield curve), demand for monetary aggregates and etc. are parameters for the model. Initial conditions will include private individuals net worth, working assets of banks etc. We use generic algorithm for the change of the parameters developed by Delli Gatti et al. [23]

The results of agents actions and interactions during analyzed period are controlled by structured variables which change and evolve in time: current account, which shows outflow of currency, employment relations, which are summed up by unemployment and wage levels, production and consumption, labor productivity, evolution of technologies and some other structured variables.

7.4 Validation of the Model

As a methodology we employ augmented Taylor rule in order to explore effects of the economy on the real equilibrium interest rate. There are several agent-based models D. Gatti et al. (2005)[23], Oeffner et al. (2008)[24] and Mandel et al. (2010)[25] analyzed Taylor rule effect on monetary policy. But we will bring new insights into the model and made the model more robust due to the integration of the globalization agents, wealth of private individuals and some other important agents into the model.

For the validation of the model we use the methodology developed by M. Guerini and A. Moneta (2016)[4]. They claim that : "For the model to be a good proxy of the data generating process, we require that it should be in a statistical equilibrium state in which the properties of the analyzed series are constant. In particular we require that the series, or a transformation of them (e.g. first differences), have distributional properties that are time-independent; secondly we require that the series are ergodic, meaning that the observed time series are a random sample of a multivariate stochastic process." This procedure validate a simulation model by the following principle: "first it estimates both the causal structure incorporated in the model (using the data artificially generated by the model) and the causal structure underlying the real-world data. Secondly, it compares the two inferred causal structures".

In addition we use standard ARCH processes in order to provide a benchmark for the explanatory power of the proposed models and to reveal the best model.Transform results of the work above into the simulation framework

The final step is to create agent-based model for the research based on theory, empirical studies and existing models. According to Richiardi et al.: "The potential lies in the modular nature of the models, which abandons restrictions of the rational expectations and embraces an evolutionary perspective"[26]. This paper "envisages the foundation of a Modular Macroeconomic Science, where new models with heterogeneous interacting agents, endowed with partial information and limited computational ability, can be created by recombining and extending existing models in a unified computational framework". The model has features that "enable it to generate gradual responses of the macroeconomic variables to the wide range of exogenous shocks"[27]. In our modeling we will use already existing algorithms. Evolution of agent algorithms and modularity will allow different research groups to develop, refine and integrate models into new models as in a Lego construction game and drive productivity through division of labor. Programmer will use source code to specify agent-based model.

For the simulation of monetary policy and equilibrium interest rate we use the AnyLogic dynamics modeling framework adopted to the equilibrium interest rate modeling.AnyLogic is a multi method simulation modeling tool developed by The AnyLogic Company. It supports agent-based, discrete event, and system dynamics simulation methodologies.We have developed an adaptation model with interconnections and interdependencies of financial and real sectors by financial flows. These free flows create imbalances and can create recessions and crises. All of these events will be forecasted by the model.

Equilibrium interest rate adjustments will be analyzed in the global environment on a decentralized ground. The decentralized adaptation model was described by Pascal Seppecher et al.[28]

8 CONCLUSION - COMPARATIVE ASSESSMENT OF MODELS BASED ON MATHEMATICAL APPROACH AND ON CAUSAL SEARCH ALGORITHMS

The linear models suffer from their ability to replicate macroeconomic reality. These models can not conceive the paradox of low interest rate, low inflation and low output. The acknowledgement of such limitations has stimulated application of agent-based models in monetary economics. These models take into account heterogeneity of agents, exogenous shocks and partially escape rationality.

Source code is the human readable instructions that a programmer writes

Table 8. Evaluation of the model developed with mathematical approach

Determinant residual covariance (adj. degrees of freedom)	0.7014
Determinant residual covariance	0.1204
Log likelihood	-1707
Akaike information criterion	29.491
Schwarz criterion	34.125

Table 9. Evaluation of the model based on casual search algorithm

Determinant residual covariance (adj. degrees of freedom)	6.412
Determinant residual covariance	4.810
Log likelihood	207.7
Akaike information criterion	-2.643
Schwarz criterion	-1.849

We use the powerful Akaike Information Criteria (AIC) tool to compare the linear model based on mathematical approach and the model based on casual search algorithm. AIC was developed by Akaike (1973)[29] to compare different models influence on given outcome. It is obvious that the model based on casual search algorithm estimated by AIC criterion is clearly preferred to the model based on mathematical approach estimated by the same criterion. Table 8 and Table 9 (above) evaluate two models. The model with casual search algorithm with Akaike Information Criterion of -2.643 performs much better than mathematical model with AIC equal to 29.491.

Proposed model improves forecasts and in forecasting real equilibrium interest rate and federal funds rate is one of the best available tools. The model provides improvements to the Structural Vector Autoregression Model.

Replacing the traditional linear mathematical model with ABC model allows to take into account influence of the globalization factors on monetary policy efficiency. Globalization dampens influence of the Federal Reserve low interest rates on inflation and output.

ACKNOWLEDGEMENT

First and foremost I express my gratitude to Professor Kunst from Vienna University for his multiple encouragements, recommendations and comments.

COMPETING INTERESTS

Author has declared that no competing interests exist.

References

[1] Blanchard O. Where danger lurks. Finance & Development (International Monetary Fund). 2014;51(3).

[2] Fagiolo G, Roventini A. Macroeconomic policy in dsge and agent-based models. Social Science Research Network (SSRN); 2016.

[3] John B. Taylor. Discretion versus policy rules in practice. Carnegie Rochester Conference Series on Public Policy. 1993;39:195-214.

[4] Guerini M, Moneta A. A method for agent-based models validation. Institute for New Economic Thinking. 2016;42.

[5] Ljung G, Box G. On a measure of lack of fit in time series models. Biometrika. 1978;65:297-303.

[6] Dickey D, Fuller W. Distribution of the estimators for autoregressive time series with a unit root. Journal of fhe American Statistical Associafion. 1970;74:427-431.

[7] Rubio-Ramirez J, Waggoner D, Zha T. Structural vector autoregressions: Theory of identification and algorithms for inference. Review of Economic Studies. 2010;77:665-696.

[8] Davies G. Who is right about the equilibrium interest rate? Financial Times; 2015.

[9] Williams JC. The decline in the natural rate of interest. Business Economics. 2015;50(2):57.

[10] Hamilton D, Harris ES, Hatzius J, West KD. The equilibrium real funds rate: Past, present and future. NBER Working Paper No. 21476; 2015.

[11] Sims C. The role of models and probabilities in the monetary policy process. Brooking Institution, Brooking Papers on Economic Activity; 2002.

[12] Cook DA, William TG. Three scenarios for interest rates in the transition to normalcy. Working paper of Federal Reserve Bank of St. Louis. First quarter Review. 2015;1-24.

[13] Borio C, Disyatat P. Global imbalances and the financial crisis: Link or no link? BIS Working Papers No 346; 2011.

[14] Chung K, Lee JE, Loukoianova E, Park H, Shin HS. Global liquidity through the lens of monetary aggregates. IMF Working paper 14/9; 2014.

[15] Bernanke B, Kuttner KN. What explains the stock markets' reaction to federal reserve policy? The Journal of Finance 60, No. 3. 2005;1221-1257.

[16] Bjornland HC, Leitemo K. Identifying the interdependence between us monetary policy and the stock market. Bank of Finland Research Discussion Paper 17; 2004.

[17] Rogoff K. Globalization and global disinflation. Economic Review, Federal Reserve Bank of Kansas City, issue QIV. 2003;45-76.

[18] Friedman M. The optimum quantity of money. McMillan; 1969.

[19] Carpenter SB, Lange B. Money demand and equity markets. Board of Governors of the Federal Reserve System and Cornerstone Research; 2002.
Available: http://www.federalreserve.gov /pubs/feds/2003/200303/200303pap.pdf

[20] Belke A, Polleit T. Monetary economics in globalized financial markets. Springer -Verlag Berlin Heidelberg. 2009;93.

[21] Ouliaris S, Pagan A, Restrepo J. Quantitative macroeconomic modeling with structural vector autoregressions - an eviews implementation. EViews online book; 2016.

[22] Spirtes P, Glymour C, Scheines R. Causation, Prediction and Search (Adaptive Computation and Machine Learning). Massachusetts Institute of Technology; 2000.

[23] Gatti D, Gaffeo D, Gallegati M, Palestrini A. The apprentice wizard: Monetary policy, complexity and learning. New Mathematics and Natural Computation. 2005;(1):109-128.

[24] Oeffner M. Agent based keynesian macroeconomics. an evolutionary model embedded in an agent based computer simulation. MPRA (Paper 18199); 2008.

[25] Mandel A, Jaeger C, Fuerst S, Lass W, Lincke D, Meissner F. Pablo-Marti F, Wolf S. Agent- based dynamics in disaggregated growth models. CES, (Working Paper 2010.77); 2010.

[26] Richiardi M, Alberto C. The future of agent -based modelling. Institute for New Economic Thinking and Nuffield College, Oxford, United Kingdom; 2015.

[27] Brayton F, Laubach T, Reifschneider D. The frb/us model: A tool for macroeconomic policy analysis. Federal Reserve working paper; 2015.

[28] Seppecher P, Salle I, Lang D. Is the market really a good teacher? market selection, collective adaptation and financial instability. Working paper of the Centre Economie de Paris Nord. CEPN; 2016.

[29] Akaike H. Information theory as an extension of the maximum likelihood principle. Second International Symposium on Information Theory, Akademiai Kiado, Budapest; 1973.

Peer-review history:
The peer review history for this paper can be accessed here:
http://sciencedomain.org/review-history/18112